

Math/Csci 2610 Fall 2004

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Exam 1

Date: Sep. 21, 2003

Duration: 50 min

Direction: For full credit, write down your arguments as clearly as possible and in an organized way, (so that also it can be used to give a partial credit).

Name: _____

Question#1: (16pts) Answer the followings:

1. (8 pts) Find out the truth value of $(p \wedge q) \oplus (p \vee q)$, when p is known to be true and q is known to be false.

2. (8 pts) By using only the logical identities, show that $p \wedge (p \rightarrow q) \equiv q \wedge p$

Question#2: (10pts)

If we will have an exam, then there will not be any tornado warning.

1. (5 pts) Write the converse of this statement.
2. (5 pts) Write the contrapositive of this statement.

Question#3: (13pts)

1. (5 pts) Let $M(x)$ be the statement x loves Mathematics.
Let $F(x,y)$ be the statement x and y are friends
Assume that the universe of discourse consists of all students with computer science major. Translate the following into English
 $\forall x \exists y (M(x) \vee (M(y) \wedge F(x,y)))$.
2. (5 pts) Write the negation of the statement $\exists x \forall y (P(x))$.
3. (3 pts) Find the truth value of $P(4)$ where $P(x)$ is given as $x^2 - 25 = 0$.

Question#4: (15pts) Let

(i) n is an even integer.

(ii) $3n + 2$ is an even.

1. (7 pts) Prove that (i) \rightarrow (ii).

2. (7 pts) Prove that (ii) \rightarrow (i).

(1 pts) After these two, what is the conclusion about (i) and (ii)

Question#5: (16pts) Let $A = \{a, b, c, d, e\}$ $B = \{1, 2, a, 3, c\}$ $C = \{2, b, a, 1\}$

1. (4 pts) Write the set $A - C$.
2. (4 pts) Write $A \cap (B \cup C)$.
3. (4 pts) Write $(A \cap B) \times (A \cap C)$
4. (4 pts) Suppose that $|A| = 4$, (cardinality of A is 4), $|B| = 7$ and $|A \cap B| = 3$.
Find $|A \cup B|$

Question#6: (10pts)

Let A and B be given two sets.

Without using Venn diagrams, show that $A \cap (A \cup B) = B - A$.

Question#7: (15pts) Let $f:A \rightarrow B$ $\forall b \in B \exists a \in A (f(a) = b)$ is true, we say that the function is surjective.

1. (3 pts) By using quantifiers, as above, state the definition of injectivity for this function $f:A \rightarrow B$

Now suppose $A=B=\text{set of integers}$ and f is given by $f(x)=4x+7$

2. (6 pts) Is f injective? Give a complete argument supporting your answer.

3. (6 pts) Is f surjective? Give a complete argument supporting your answer.

Question#8: (5pts) Let $F: B^3 \rightarrow B$, a Boolean function of degree 3, be given by $F(x)= x +yz$
Evaluate the followings: (Show your work step by step!)

1. $F(1,0,1)$

2. $F(0,1,1)$

Question#9: (5pts) (Bonus) Show that $((p \vee q) \wedge (p \rightarrow q)) \rightarrow q$, is a tautology.

Question#10: (5pts) (Bonus) For any real numbers x , show that the following equality holds.

$$\lfloor 3x \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{3} \rfloor + \lfloor x + \frac{2}{3} \rfloor$$