

Math/Csci 2610 Fall 2004

Instructor: Zubeyir Cinkir

# Exam 1

Date: Sep. 16, 2003

Duration: 50 min

Direction: For full credit, write down your arguments as clearly as possible and in an organized way, (so that also it can be used to give a partial credit).

Name: \_\_\_\_\_

**Question#1:** (20pts) Answer the followings:

1. (10 pts) Show that  $(p \oplus q) \rightarrow (p \vee q)$  is a tautology by using truth table.

2. (10 pts) By the logical identities, show that the propositional compound  $(\neg q \wedge p) \rightarrow (p \vee q)$  is a tautology.

**Question#2:** (10pts) **If It is rainy, then I will stay home.**

1. (5 pts) Write the converse of this statement.
2. (5 pts) Write the contrapositive of this statement.

**Question#3:** (10pts)

1. (5 pts) Let **A(x)** be the statement **x lives in Atlanta**  
Let **F(x,y)** be the statement **x and y are friends**  
Assume that the universe of discourse consists of all students at UGA. Translate the following into English  
 $\forall x \exists y (A(x) \vee (A(y) \wedge F(x, y)))$ .
2. (5 pts) Write the negation of the statement in 1. Then, translate it into English.

**Question#4:** (15pts) Let

(i)  $n$  is an odd integer.

(ii)  $5n + 4$  is an odd.

1. (7 pts) prove that (i)  $\rightarrow$  (ii).

2. (7 pts) prove that (ii)  $\rightarrow$  (i).

(1 pts) After these two, what is the conclusion about (i) and (ii)

**Question#5:** (18pts) Let  $A = \{a, b, c, d, e\}$      $B = \{1, a, b, 2, c\}$      $C = \{3, c, a, 1, f\}$

1. (3 pts) Write the set  $A \cap B$ .
2. (3 pts) Write  $A \cup B$ .
3. (3 pts) Write  $A - (B \cap C)$
4. (3 pts) Write  $(A \cap C) \times (C - B)$
5. (3 pts) What is the cardinality of  $P(A)$ , power set of  $A$ ?
6. (3 pts) What is the cardinality of  $A \times C$ ?

**Question#6:** (10pts) Let  $A$  and  $B$  be given two sets. Show that  $A \cup (B - A) = A \cup B$ .

**Question#7:** (17pts) Let  $f:A \rightarrow B$   $\forall x \in A \forall y \in A ((f(x) = f(y)) \rightarrow x = y)$  is true, we say that the function is injective.

1. (3 pts) By using quantifiers, as above, state the definition of surjectivity for this function  $f:A \rightarrow B$

Now suppose  $A=B=\text{set of integers}$  and  $f$  is given by  $f(x)=5x-3$

2. (7 pts) Is  $f$  injective? Give a complete argument supporting your answer.

3. (7 pts) Is  $f$  surjective? Give a complete argument supporting your answer.

**Question#8:** (10pts) (Bonus) For any real numbers  $x$ , show that the following equality holds.

$$\lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{2} \rfloor$$

**Question#9:** (5pts) (Bonus) Write the negation of the following statement  
 $\forall x(P(x)) \rightarrow \forall x\exists y(P(x) \wedge \neg Q(y))$