

Math/Csci 2610 Fall 2004

Instructor: Zubeyir Cinkir

FINAL EXAM

Date: Dec. 14, 2003

Duration: 150 min

Direction: For full credit, write down your arguments as clearly as possible and in an organized way, (so that also it can be used to give a partial credit).

Name: _____

Question#1: (6pts) Answer the followings:

1. (3 pts) What is the truth table for $(p \wedge q) \rightarrow (p \vee q)$.

2. (3 pts) By using only the logical identities, show that $(p \rightarrow q) \vee (\neg q)$ is a tautology.

Question#2: (6pts) Answer the followings:

1. (3 pts) Let m and n be two odd integers. Show that $m.n$ is also an odd integer.

2. (3 pts) What is the negation of the statement $\exists \mathbf{x} \mathbf{P}(\mathbf{x}) \rightarrow \forall \mathbf{x} (\mathbf{P}(\mathbf{x}) \wedge \mathbf{Q}(\mathbf{x}))$.

Question#5: (4pts) Find as good big-O estimate as possible to the following

1. (2 pts) $(n^2 + 5n + 100)(4n^3 + 70n + 3)$

2. (2 pts) $25n \log n + 300 + (3n + 1)(2n + 5)$

Question#6: (7pts) Answer the followings:

1. (3 pts) Find the prime factorization of 660

2. (4 pts) Let $a=2730$ and $b=570$. Find $\gcd(a,b)$ by using Euclidean algorithm.

Question#7: (10pts) Answer the followings:

1. (3 pts) Let m be a positive integer. Prove that
If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$ then $a + c \equiv b + d \pmod{m}$

2. (3 pts) Find the decimal expansion of $(321)_4$.

3. (4 pts) Find the two smallest positive solutions to the system of congruences
 $x \equiv 2 \pmod{3}$.
 $x \equiv 1 \pmod{4}$.
 $x \equiv 2 \pmod{5}$.

Question#8: (6pts) Let $A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 1 \\ 2 & 0 \end{bmatrix}$. Do the following matrix operations

1. (3 pts) $A+B$

2. (3 pts) $A \cdot B$

Question#9: (4pts) Prove that there is no integers x and y that are solutions to the equation $3x^2 + 5y^2 = 30$.

Question#10: (7pts) Answer the followings:

1. (3 pts) Evaluate $\sum_{i=1}^7 2i$

2. (4 pts) By using the mathematical induction, prove that $\sum_{j=1}^n j = \frac{n(n+1)}{2}$.

Question#11: (4pts) How many positive integers less than 1000 is divisible by 5 but not by 7.

Question#12: (4pts) A particular brand of shirt comes in 5 colors, has a male and female version, and comes in three sizes for each sex. How many different types of this shirts are made?

Question#13: (4pts) Compute the followings:

1. (2 pts) $C(8,5)$

2. (2 pts) $P(7,3)$

Question#14: (4pts) How many positive integers that are divisible by 5 and having four distinct digits are there?

Question#15: (4pts) How many integer solutions are there to the equation $x_1 + x_2 + x_3 = 14$ where $x_1 \geq -2$, $x_2 \geq 4$ and $x_3 \geq -1$.

Question#16: (4pts) What is the probability that the sum of the numbers on two dice is even when they are rolled?

Question#17: (4pts) Suppose that **E** and **F** are independent events in a sample space **S**. Given that $p(E) = \frac{2}{3}$, $p(F) = \frac{1}{5}$. Compute $p(E \cup F)$.

Question#18: (4pts) A biased coin is flipped three times. Let X be the random variable that assigns to an outcome the number of heads in this outcome. Assume that the probability of head come up is four times as much as the probability of tail comes up. What is the expected value of X ?

Question#19: (6pts) Let $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2$, $a_0 = 2$, and $a_1 = 6$. Answer the followings:

1. (2 pts) Find a_3 and a_4 .
2. (2 pts) Write the related characteristic equation.
3. (2 pts) What is the general solution of the recurrence relation.

Question#20: (4pts) (Bonus) Prove that there is no integers x and y that are solutions to the equation $3x^2 - 8y^2 = 41$.

Question#21: (4pts) (Bonus) Show that these two statements are equivalent.

(i) x is irrational

(ii) $5x+1$ is irrational

Question#22: (4pts) (Bonus) Let m and n be two odd integers. prove by contradiction that if $m.n$ is an odd integer, then both m and n are odd integers.