QUIZ-9
Name:

Math/CSCI 2610 Fall 2004
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Please show all your work!

1. (1 point) How many integers between 1000 and 9999 (inclusive) are divisible by 5 but not 7?

Let $A = \{ n \mid 1000 \le n \le 9999 \text{ and } 5 \mid n \}$ - we want $|A-B|$.

$B = \{ n \mid 1000 \le n \le 9999 \text{ and } 7 \mid n \}$ Note that $|A-B| = |A| - |A \cap B|$

$|A| = \left\lfloor \frac{9999-1000+1}{5} \right\rfloor = 1800$ ← incl.

$A \cap B = \{ n \mid 1000 \le n \le 9999,\ 5 \mid n \text{ \& } 7 \mid n \}$ so $|A \cap B| = \left\lfloor \frac{9999-1000}{35} \right\rfloor = 257$ ← since inclusive

$= \{ n \mid 1000 \le n \le 9999 \text{ and } 35 \mid n \}$ Hence, $|A-B| = 1800 - 257 = 1543$

2. (1 point) Let n be a positive integer. Show that in any set of n consecutive integers there is exactly one divisible by n.

$R = \{0, 1, 2, 3, \ldots, n-1\}$ is the set of possible residue classes modulo n. Note $|R| = n$. Also, for any n consecutive integers, say $a+1, a+2, a+3, \ldots, a+n$, we have $\{ a+i \pmod n \mid i = 1, 2, \ldots, n \} = R$

Hence, $a + k \equiv 0 \pmod n$ for some $k \in \{1, 2, \ldots, n\}$, i.e. $n \mid (a+k)$.

3. (1 point) What is the coefficient of $x^8 y^9$ in the expansion of $(3x + 2y)^{17}$.

$C \binom{17}{9} (3x)^{17-9} (2y)^9$ is the term with $x^8 y^9$ in the expansion of $(3x+2y)^{17}$

so the coef. is $C \binom{17}{9} \cdot 3^8 \cdot 2^9 = \frac{17!}{9! \cdot 8!} \cdot 3^8 \cdot 2^9$