Section 2.2 pg 73 # 9, 13, 17, 19, 25, 29, 31, 33, 37, 41, 43, 45

(9) $\lim_{x\to 1} \sqrt{4x+5} \underset{\text{composition}}{=} \sqrt{\lim_{x\to 1} 4x+5} = \sqrt{9} = 3$

(13) $\lim_{z\to 8} \dfrac{z^{2/3}}{z-\sqrt{2z}} \underset{\text{quotient}}{=} \dfrac{\lim_{z\to 8} z^{2/3}}{\lim_{z\to 8} z-\sqrt{2z}} \underset{\text{sum}}{=} \dfrac{\lim_{z\to 8} z^{2/3}}{\lim_{z\to 8} z - \lim_{z\to 8}\sqrt{2z}}$

$\underset{\text{composition}}{=} \dfrac{\lim_{z\to 8} z^{2/3}}{\lim_{z\to 8} z - \sqrt{\lim_{z\to 8} 2z}} \underset{\text{evaluation}}{=} \dfrac{8^{2/3}}{8-\sqrt{2\cdot 8}} = \dfrac{4}{8-4} = \dfrac{4}{4} = 1$

(17) $\lim_{x\to -2} \sqrt[3]{\dfrac{x+2}{(x-2)^2}} \underset{\text{composition}}{=} \sqrt[3]{\lim_{x\to -2} \dfrac{x+2}{(x-2)^2}} \underset{\text{quotient}}{=} \sqrt[3]{\dfrac{\lim_{x\to -2} x+2}{\lim_{x\to -2} (x-2)^2}}$

$\underset{\text{composition}}{=} \sqrt[3]{\dfrac{\lim_{x\to -2} x+2}{\left(\lim_{x\to -2} (x-2)\right)^2}} \underset{\text{evaluation}}{=} \sqrt[3]{\dfrac{0}{16}} = \sqrt[3]{0} = 0$

(19) $\lim_{x\to -1} \dfrac{x+1}{x^2-x-2} = \lim_{x\to -1} \dfrac{x+1}{(x+1)(x-2)} = \lim_{x\to -1} \dfrac{1}{x-2} = -\dfrac{1}{3}$

(25) $\lim_{z\to -2} \dfrac{(z+2)^2}{z^4-16} = \lim_{z\to -2} \dfrac{(z+2)^2}{(z^2-4)(z^2+4)} = \lim_{z\to -2} \dfrac{(z+2)^2}{(z-2)(z+2)(z^2+4)}$

$= \lim_{z\to -2} \dfrac{z+2}{(z-2)(z^2+4)} = 0$

(29) $$\lim_{x\to 3} \frac{\frac{1}{x}-\frac{1}{3}}{x-3} = \lim_{x\to 3} \frac{\frac{3-x}{3x}}{x-3} = \lim_{x\to 3} \frac{-(x-3)}{3x}\cdot\frac{1}{x-3}$$

$$= \lim_{x\to 3} \frac{-1}{3x} = \frac{-1}{9}$$

(31) $$\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \lim_{x\to 4} \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{\sqrt{x}-2} = \lim_{x\to 4} \sqrt{x}+2 = \sqrt{4}+2 = 4$$

(33) $$\lim_{t\to 0} \frac{\sqrt{t+4}-2}{t} = \lim_{t\to 0} \frac{(\sqrt{t+4}-2)(\sqrt{t+4}+2)}{t(\sqrt{t+4}+2)} = \lim_{t\to 0} \frac{t+4-4}{t(\sqrt{t+4}+2)}$$

$$= \lim_{t\to 0} \frac{1}{\sqrt{t+4}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{4}$$

(37) $f(x) = x^3$

$$m(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h\to 0} \frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$$

$$= \lim_{h\to 0} \frac{3x^2h+3xh^2+h^3}{h} = \lim_{h\to 0} 3x^2+3xh+h^2 = 3x^2$$

where $x=2$, $f(2) = 2^3 = 8$, $m(2) = 3\cdot 2^2 = 3\cdot 4 = 12$

So the equation of the tangent line is $y-8 = 12(x-2)$

$$y-8 = 12x-24$$

$$y = 12x-16$$

(41) $f(x) = \dfrac{2}{x-1}$

$$m(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{2}{x+h-1} - \frac{2}{x-1}}{h} = \lim_{h\to 0} \frac{\frac{2(x-1) - 2(x+h-1)}{(x-1)(x+h-1)}}{h}$$

$$= \lim_{h\to 0} \frac{(2x-2-2x-2h+2)}{(x-1)(x+h-1)h} = \lim_{h\to 0} \frac{-2h}{(x-1)(x+h-1)h}$$

$$= \lim_{h\to 0} \frac{-2}{(x-1)(x+h-1)} = \frac{-2}{(x-1)^2}$$

$$(2, f(2)) = \left(2, \frac{2}{2-1}\right) = (2,2) \qquad m(2) = \frac{-2}{(2-1)^2} = -2$$

Equation of tangent line

$$y-2 = -2(x-2)$$
$$y-2 = -2x+4$$
$$y = -2x+6$$

(43) $f(x) = \dfrac{1}{\sqrt{x+2}}$ $\qquad m(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \dfrac{\frac{1}{\sqrt{x+h+2}} - \frac{1}{\sqrt{x+2}}}{h}$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\sqrt{x+2} - \sqrt{x+h+2}}{\sqrt{x+2}\,\sqrt{x+h+2}}\right] = \lim_{h\to 0} \frac{(\sqrt{x+2} - \sqrt{x+h+2})(\sqrt{x+2} + \sqrt{x+h+2})}{h\sqrt{x+2}\,\sqrt{x+h+2}\,(\sqrt{x+2} + \sqrt{x+h+2})}$$

$$= \lim_{h\to 0} \frac{(x+2) - (x+h+2)}{h(x+2)\sqrt{x+h+2} + h(x+h+2)\sqrt{x+2}} = \lim_{h\to 0} \frac{-h}{h(x+2)\sqrt{x+h+2} + h(x+h+2)\sqrt{x+2}}$$

$$= \lim_{h\to 0} \frac{-1}{(x+2)\sqrt{x+h+2} + (x+h+2)\sqrt{x+2}} = \frac{-1}{(x+2)\sqrt{x+2} + (x+2)\sqrt{x+2}} = \frac{-1}{2(x+2)^{3/2}}$$

$$(2, f(2)) = \left(2, \frac{1}{\sqrt{2+2}}\right) = \left(2, \frac{1}{4}\right) \qquad m(2) = \frac{-1}{2(4)^{3/2}} = \frac{-1}{2\cdot 8} = \frac{-1}{16}$$

So equation of tangent line is

$$y - \frac{1}{4} = \frac{-1}{16}(x-2) \qquad \to \quad y = \frac{-1}{16}x + \frac{3}{8}$$

$$y - \frac{1}{4} = \frac{-1}{16}x + \frac{1}{8}$$

(45) $f(x) = \sqrt{2x+5}$

$$m(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{2(x+h)+5} - \sqrt{2x+5}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{2(x+h)+5} - \sqrt{2x+5}}{h} \cdot \frac{\sqrt{2(x+h)+5} + \sqrt{2x+5}}{\sqrt{2(x+h)+5} + \sqrt{2x+5}}$$

$$= \lim_{h\to 0} \frac{2(x+h)+5 - (2x+5)}{h\left(\sqrt{2(x+h)+5} + \sqrt{2x+5}\right)} = \lim_{h\to 0} \frac{2x+2h+5-2x-5}{h\left(\sqrt{2(x+h)+5} + \sqrt{2x+5}\right)}$$

$$= \lim_{h\to 0} \frac{2}{\sqrt{2(x+h)+5} + \sqrt{2x+5}} = \frac{1}{\sqrt{2x+5}}$$

$(2, f(2)) = (2, \sqrt{2\cdot 2+5}) = (2, \sqrt{9}) = (2, 3)$ $\quad m(2) = \frac{1}{\sqrt{2\cdot 2+5}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$

Equation of tangent line $\quad y - 3 = \frac{1}{3}(x-2)$

$$y - 3 = \frac{1}{3}x - \frac{2}{3}$$

$$y = \frac{1}{3}x + \frac{7}{3}$$