section 2.4
pg 97

# 1, 3, 5, 7, 9, 11, 15, 19, 25, 33, 35

① $f(x) = 2x^5 + 7x^2 + 13$. This is a polynomial and polynomials are continuous everywhere

③ $g(x) = \dfrac{2x-1}{4x^2+1}$ This is a rational function, and they are continuous everywhere <u>except</u> where the denominator is zero. But $4x^2+1 \neq 0$ for any x, so the function is continuous everywhere

⑤ $h(x) = \sqrt{x^2+4x+5}$ notice that the function $x^2+4x+5$ is a polynomial which is continuous everywhere. It is also never negative. The function $\sqrt{x}$ is continuous for all $x \geq 0$. So by composition $h(x)$ is continuous for all x.

⑦ $f(x) = \dfrac{1-\sin x}{1+\cos^2 x}$ The numerator and denominator are always continuous. Now check the denominator for zeros. $1+\cos^2 x = 0$ $\cos^2 x = -1$. Can't happen since $\cos^2 x \geq 0$ for all x. So $f(x)$ is continuous everywhere.

⑨ $f(x) = \dfrac{1}{x+1}$ $x > -1$. Take $a > -1$. Then $\lim\limits_{x \to a} f(x) = \lim\limits_{x \to a} \dfrac{1}{x+1} = \dfrac{1}{a+1}$

and $f(a) = \dfrac{1}{a+1}$. So $f(x)$ is continuous when $x > -1$. This is <u>not</u> true on the interval $x \geq -1$ since $f(x)$ is not defined when $x = -1$

(11) $g(t) = \sqrt{9-4t^2} \qquad -3/2 \le t \le 3/2.$

Note that $9-4t^2 \geq 0$ when $-3/2 \le t \le 3/2$, so for $-3/2 < a < 3/2$

$\lim_{t \to a} \sqrt{9-4t^2} = \sqrt{9-4a^2}$ and $g(a) = \sqrt{9-4a^2}$. Now check the endpoints.

$$\lim_{t \to -3/2^+} \sqrt{9-4t^2} = \sqrt{\lim_{t \to -3/2^+} 9-4t^2} = \sqrt{0} = 0 \qquad f(-3/2) = 0$$

$$\lim_{t \to +3/2^-} \sqrt{9-4t^2} = \sqrt{\lim_{t \to 3/2^-} 9-4t^2} = \sqrt{0} = 0 \qquad f(3/2) = 0.$$

So $g(t)$ is continuous on $-3/2 \le t \le 3/2$

(15) $f(x) = 2x + \sqrt[3]{x}$. This function is well defined everywhere so it is continuous everywhere

(19) $f(x) = \dfrac{1}{x^2+1}$ This function is well defined everywhere so it is continuous everywhere

(25) $f(x) = \sqrt[3]{\dfrac{x+1}{x-1}}$ This function is continuous everywhere except where $x-1=0$, ie when $x=1$.

(33) $f(x) = \dfrac{1}{\sin 2x}$ This function is continuous except where $\sin 2x = 0$. Since $\sin(n\pi) = 0$ where $n$ is an integer $(0, \pm 1, \pm 2, \dots)$ it is not continuous when $n\pi = 2x$, or $x = \dfrac{n\pi}{2}$

(35) $f(x) = \sin|x|$ This is a composition of continuous functions so it is continuous everywhere.