Section 3.1 pg 112 # 21, 23, 25, 27, 29, 39, 41, 42, 52, 53

21 $x = 100 - 16t^2$ position.

velocity $\frac{dx}{dt} = -16 \cdot 2 \cdot t = -32t$

velocity $= 0$ when $-32t = 0$, $t = 0$.

So position is $x(0) = 100 - 16 \cdot 0^2 = 100$

23 $x = -16t^2 + 80t - 1$ position

velocity $\frac{dx}{dt} = -32t + 80$ velocity $= 0$ when $-32t + 80 = 0$

$-32t = -80$

$t = \frac{80}{32} = 5/2$

So position is $x(5/2) = -16\left(\frac{5}{2}\right)^2 + 80\left(\frac{5}{2}\right) - 1$

$= -16 \cdot \frac{25}{4} + 40 \cdot 5 - 1$

$= -4 \cdot 25 + 200 - 1$

$= -100 + 200 - 1$

$= 99$

25 $x = 100 - 20t - 5t^2$ position

velocity $\frac{dx}{dt} = -10t - 20$ velocity $= 0$ when $-10t - 20 = 0$

$-10t = 20$

$t = -2$

So position is $x(-2) = 100 - 20(-2) - 5(-2)^2$

$= 100 + 40 - 20$

$= 120$

27 $y = -16t^2 + 64t$ max height happens when velocity $= 0$

velocity $\frac{dy}{dt} = -32t + 64$ $-32t + 64 = 0$

$-32t = -64$

$t = 2$

max height $y(2) = -16(2)^2 + 64(2) = -16 \cdot 4 + 128 = -64 + 128 = 64$ feet

(29) $y = -16t^2 + 96t + 50$

velocity $\frac{dy}{dt} = -32t + 96$ velocity $= 0$ $-32t + 96 = 0$

$-32t = -96$

max height $t = 3$

$y(3) = -16 \cdot 9 + 96 \cdot 3 + 50 = -144 + 288 + 50 = 194$ feet

(39) position $x(t) = 100t - 5t^2$. when the car is stopped, the velocity is zero

velocity $\frac{dx}{dt} = -10t + 100$ $-10t + 100 = 0$

$-10t = 100$

$t = 10$

so it stops in 10 seconds the position is

$x(10) = 100 \cdot 10 - 5(10)^2 = 1000 - 500 = 500$ ft.

(41) $P(t) = 100[1 + .3t + .04t^2] = 100 + 30t + 4t^2$

initial size $P(0) = 100$. Doubled in size means when $P(t) = 200$.

Now find t.

$$100 + 30t + 4t^2 = 200$$

$$-100 + 30t + 4t^2 = 0 \quad \text{(divide by 2)}$$

$$-50 + 15t + 2t^2 = 0$$

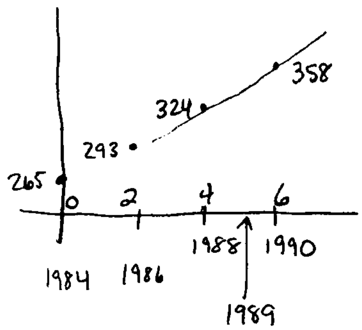
$$t = \frac{-15 \pm \sqrt{15^2 - 4(-50)(2)}}{2 \cdot 2} = \frac{-15 \pm \sqrt{625}}{4} = \frac{-15 \pm 25}{4}$$

$$= -10, \ 5/2. \quad \text{(can't use } -10)$$

So it takes 2.5 years for the population to double

Rate of growth is $P'(t) = 30 + 8t$. Rate of growth when $P(t) = 200$ is $P'(5/2) = 30 + 8(5/2) = 30 + 4 \cdot 5 = 50 \ \frac{\text{chipmunks}}{\text{month}}$

(42)



Find slope of line between years 4 and 6

$m = \frac{358 - 324}{2} = \frac{34}{2} = 17$

So the estimate of growth rate in 1989 is

$17 \frac{\text{thousands of people}}{2 \text{ years}}$

(52) $P(t) = 100\left[1 + .04t + .003t^2\right] = 100 + 4t + .3t^2$

$P'(t) = 4 + .6t$

a) in 1986 it is year $t = 6$ (since $t=0$ is 1980)

$P'(6) = 4 + .6 \cdot 4 = 6.4 \frac{\text{thousands of people}}{\text{year}}$

b) average rate of change between 1983 $(t=3)$ to 1988 $(t=8)$

$P(3) = 100 + 4 \cdot 3 + .3(3)^2 = 114.7$

$P(8) = 100 + 4 \cdot 8 + .3 \cdot (8)^2 = 151.2$

So average rate of change is $\frac{151.2 - 114.7}{8-3} = 7.3 \frac{\text{thousands of people}}{\text{year}}$

(53) $P(t) = 10 + t - .1t^2 + .006t^3$

$P'(t) = 1 - .2t + .018t^2$

Average rate of change between 0 & 10

is $\frac{P(10) - P(0)}{10 - 0} = \frac{10 + 10 - .1(100) + .006(10)^3 - 10}{10}$

$= 6/10 = 3/5 \frac{\text{thousands of people}}{\text{year}}$

We want instantaneous to equal average. ie $P'(t) = 3/5$

$1 - .2t + .018t^2 = 3/5$

$.4 - .2t + .018t^2 = 0$

$t = \frac{+.2 \pm \sqrt{.2^2 - 4(.018)(.4)}}{2(.018)}$

$= \frac{+.2 \pm .106}{.036} = 8.5, 2.61$

So when $t = 8.5$ years or $t = 2.61$ years

ie in 1998.5 and 1992.61

the instantaneous and average rate of change are the same.