section 3.2

pg 123 # 5, 17, 51, 53, 55, 57, 59, 62, 63, 65

prove linear combination law (proof on pg 117 in text)

(5) $h(x) = (x+1)^3$ (multiply out then take derivative & factor)

$h'(x) = 3(x+1)^2$

(17) $g(y) = 2y(3y^2-1)(y^2+2y+3)$

$= (6y^3-2y)(y^2+2y+3)$

$= 6y^5+12y^4+18y^3-2y^3-4y^2-6y$

$= 6y^5+12y^4+18y^3-4y^2-6y$

$g'(y) = 30y^4+48y^3+54y^2-8y-6$

(51) $V = V_0\left[1-(6.427\times10^{-5})T+(8.505\times10^{-6})T^2-(6.790\times10^{-8})T^3\right]$

$V_0 = 1000$

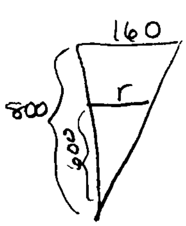
$V = 1000 - .06427T + .008505T^2 - .0000679T^3$

$V' = -.06427 + .01701T - .0002037T^2$

when $T=0$, $V'(0) = -.06427$, so initially it is contracting at a rate of $-.06427$ cm³/s

(53) Volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

$\frac{dV}{dh} = \frac{1}{3}\pi r^2$ when $h=600$, we need to find the radius



we can use properties of similar triangles

$\frac{160}{800} = \frac{r}{600}$

$r = \frac{160\cdot600}{800} = \frac{6\cdot160}{8} = 120$

so $\frac{dV}{dh} = \frac{1}{3}\pi(120)^2 = \frac{14400\pi}{3}$ cm³/cm

(55) $y = x^3$

$\frac{dy}{dx} = 3x^2$. At the point (a, a^3), $\frac{dy}{dx} = 3a^2$. ← slope

equation of tangent line is $y - a^3 = 3a^2(x-a)$

through the point $(1,5)$ is then

$5 - a^3 = 3a^2(1-a)$ solve for a

$5 - a^3 = 3a^2 - 3a^3$

$2a^3 - 3a^2 + 5 = 0$ graph to find a, $a \approx -1$

so the tangent line equation is $y - (-1)^3 = 3(-1)^2(x-(-1))$

$y + 1 = 3(x+1)$

$y + 1 = 3x + 3$

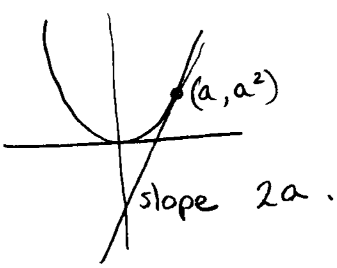
$y = 3x + 2$

(57) $y = x^2$

$y'(x) = 2x$. Given a point (a, a^2), the slope of the tangent line is $y'(a) = 2a$, and the equation of the tangent line at (a, a^2) is $y - a^2 = 2a(x-a)$

$y - a^2 = 2ax - 2a^2$

$y - 2ax = -a^2$.



For a different point (b, b^2), the slope will be $2b$, which is not the same as $2a$. So you will have 2 different tangent line equations.

(59) $n \geq 2$. $y = x^n$ $y'(x) = nx^{n-1}$. $P(x_0, y_0)$

Equation of tangent line through P is $y - y_0 = nx_0^{n-1}(x - x_0)$

We need to find the x-intercept, ie where $y = 0$.

$y - y_0 = nx_0^{n-1}x - nx_0^n$

$0 - y_0 = nx_0^{n-1}x - nx_0^n$

(ask yourself why we need $n \geq 2$)

$\frac{-y_0 + nx_0^n}{nx_0^{n-1}} = x$

↑ x intercept.

(62) $u_1 u_2 u_3 u_4 = (u_1 u_2 u_3) u_4$

Take derivative using product rule

$$(u_1 u_2 u_3 u_4)' = [(u_1 u_2 u_3) u_4]' = (u_1 u_2 u_3)' u_4 + u_1 u_2 u_3 u_4'$$

$$= [u_1' u_2 u_3 + u_1 u_2' u_3 + u_1 u_2 u_3'] u_4 + u_1 u_2 u_3 u_4'$$

$$= u_1' u_2 u_3 u_4 + u_1 u_2' u_3 u_4 + u_1 u_2 u_3' u_4 + u_1 u_2 u_3 u_4' \quad (*)$$

which verifies formula (16) for $n=4$

Now

$$(u_1 u_2 u_3 u_4 u_5)' = [(u_1 u_2 u_3 u_4) u_5]' = (u_1 u_2 u_3 u_4)' u_5 + u_1 u_2 u_3 u_4 u_5'$$

using the above equation (*)

$$= [u_1' u_2 u_3 u_4 + u_1 u_2' u_3 u_4 + u_1 u_2 u_3' u_4 + u_1 u_2 u_3 u_4'] u_5 + u_1 u_2 u_3 u_4 u_5'$$

$$= u_1' u_2 u_3 u_4 u_5 + u_1 u_2' u_3 u_4 u_5 + u_1 u_2 u_3' u_4 u_5 + u_1 u_2 u_3 u_4' u_5 + u_1 u_2 u_3 u_4 u_5'$$

verifying (16) for $n=5$

(63) Think of $(f(x))^n = \underbrace{f(x) f(x) \cdots f(x)}_{n \text{ times}}$

using equation (16),

$$D_x[(f(x))^n] = D_x[\overbrace{f(x) f(x) \cdots f(x)}^{n \text{ times}}]$$

$$= f'(x) \underbrace{f(x) \cdots f(x)}_{n-1 \text{ times}} + f(x) f'(x) \underbrace{f(x) \cdots f(x)}_{n-2 \text{ times}} + \cdots + \underbrace{f(x) \cdots f(x)}_{n-1 \text{ times}} f'(x)$$

Notice that each one has $n-1$ $f(x)$'s in it, and there are n terms so

$$= n(f(x))^{n-1} f'(x)$$

(65) $g(x) = (x^3 - 17x + 35)^{17}$

$$g'(x) = 17(x^3 - 17x + 35)^{16}(3x^2 - 17)$$