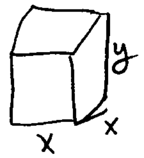
Section 3.9 pg 195 # 36, 53, 57, 68

(36)



given $\frac{dx}{dt} = 2$ in/hr

$\frac{dy}{dt} = 3$ in/hr

The rate of water flowing into the pan is the same as the amount of water melting off

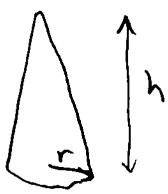
$V = x^2 y$

$$\frac{dV}{dt} = 2x\frac{dx}{dt}y + x^2\frac{dy}{dt}$$

base = 15 in
height = 20 in

$$\frac{dv}{dt} = 2(15)(2)(20) + (15)^2(3)$$
$$= 1650 \text{ in}^3/\text{hr}$$

(53)



$\frac{dh}{dt} = -3$ cm/s

$\frac{dr}{dt} = 2$ cm/s

$r = 4$cm
$h = 6$m

negative b/c decreasing

Notice that our answer for $\frac{dV}{dt}$ is positive, so the volume is increasing

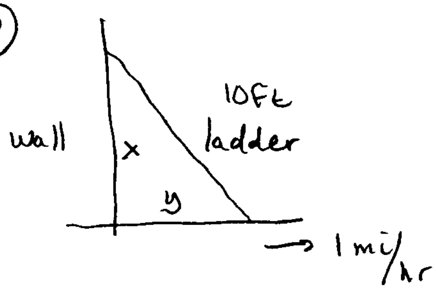
$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt} = \frac{1}{3}\pi\left[2r\frac{dr}{dt}h + r^2\frac{dh}{dt}\right]$$

$$= \frac{1}{3}\pi\left[2(4)(2)(6) + (4)^2(-3)\right]$$

$$= 16\pi \text{ cm}^3/\text{s}$$

57



$$x^2+y^2=10^2$$

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

when $x=4$,

$$y^2+4^2=10^2$$

$$y=\sqrt{10^2-4^2}=2\sqrt{21}$$

$$\frac{dy}{dt}=1\text{ mi/hr}=5280\text{ ft/hr}$$

$$=\frac{5280}{60^2}\text{ ft/s}=1.46$$

$$2(4)\frac{dx}{dt}+2\sqrt{21}(1.46)=0$$

$$\frac{dx}{dt}=\frac{-2\sqrt{21}(1.46)}{8}\approx -1.67\text{ ft/sec}$$

b) when $x=1\text{ in}=\frac{1}{12}\text{ ft}$,

$$\left(\frac{1}{12}\right)^2+y^2=10^2$$

$$y=9.99$$

$$2x\frac{dx}{dt}=-2y\frac{dy}{dt}$$

$$2\left(\frac{1}{12}\right)\frac{dx}{dt}=-2(9.99)(1.46)$$

$$\frac{dx}{dt}=-175\text{ ft/sec}$$

c) when $x=1\text{mm}\cdot\frac{1\text{in}}{2.54\text{ mm}}\cdot\frac{1\text{ft}}{12\text{in}}=.0328$

$$x^2+y^2=100$$

$$y^2=100-(.0328)^2$$

$$y=9.99\ldots$$

almost 10!

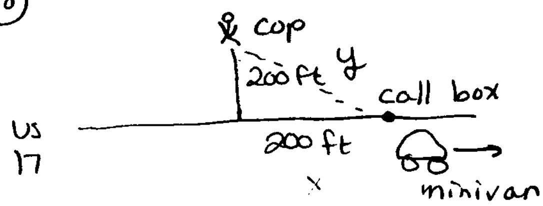
$$2x\frac{dx}{dt}=-2y\frac{dy}{dt}$$

$$2(.0328)\frac{dx}{dt}=-2(9.99)(1.46)$$

$$\frac{dx}{dt}=444\text{ ft/s}$$

It looks like that as $x\to 0$, $\frac{dx}{dt}\to\infty$, which we know is impossible.

68



distance between officer & minivan is increasing at a rate of 66 ft/sec

speed limit is $55 \frac{mi}{hr}$ (convert units!)

$$\frac{55\text{ mi}}{\text{hr}} \cdot \frac{5280\text{ft}}{1\text{mi}} \cdot \frac{1\text{hr}}{60^2\text{ sec}} = 80.67 \text{ ft/sec}$$

$\frac{dy}{dt} = 66 \text{ft/sec}$ when $x = 200$ (y is hypotenuse)

want to find $\frac{dx}{dt}$, which is speed of van

$$x^2 + 200^2 = y^2$$

when $x = 200$, $200^2 + 200^2 = y^2$

$y = 282.8$

$$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$$

$$2(200)\frac{dx}{dt} = 2(282.8)(66)$$

$\frac{dx}{dt} = 93.3$ ft/s, which is faster than the speed limit of 80.67 ft/sec.

The difference is 12.6 ft/s. convert to mi/hr

$$12.6\frac{\text{ft}}{\text{s}} \cdot \frac{60^2\text{s}}{1\text{hr}} \cdot \frac{1\text{mi}}{5280\text{ ft}} = 8.6 \text{ mi/hr}$$

so he is going more than 5 mi/hr over the speed limit. Give the minivan a ticket!