Section 2.1 pg 62 # 5, 7, 8, 9, 11, 19, 21, 28, 33, 34

(5) $f(x) = 4x - 5$. Note that this is the equation of a line with slope 4. So $m(a) = 4$ for any point $a = x$. It is its own tangent line, so equation of tangent line at $(2, f(2))$ is $y = 4x - 5$.

(7) $f(x) = 2x^2 - 3x + 4$

using slope predictor $m(x) = 2px + q$ gives

$m(x) = 2(2)x - 3 = 4x - 3$. when $x = a$, $m(a) = 4a - 3$.

At the point $(2, f(2)) = (2, 2 \cdot 4 - 3 \cdot 2 + 4) = (2, 6)$ is

$y - 6 = m(2)(x - 2)$ or

$$y - 6 = (4 \cdot 2 - 3)(x - 2)$$
$$y - 6 = 5(x - 2)$$
$$y = 5x - 10 + 6$$
$$y = 5x - 4.$$

(8) $f(x) = 5 - 3x - x^2$. Putting in traditional form, $f(x) = -x^2 - 3x + 5$

$m(x) = 2(-1)x - 3 = -2x - 3$. When $x = a$, $m(a) = -2a - 3$

At the point $(2, f(2)) = (2, -4 - 3 \cdot 2 + 5) = (2, -5)$ is

$y + 5 = m(2)(x - 2)$ or

$$y + 5 = (-2 \cdot 2 - 3)(x - 2)$$
$$y + 5 = (-7)(x - 2)$$
$$y + 5 = -7x + 14$$
$$y = -7x + 9$$

(9) $f(x) = 2x(x + 3)$, or $f(x) = 2x^2 + 6x$

$m(x) = 2(2)x + 6 = 4x + 6$ when $x = a$, $m(a) = 4a - 6$

At $(2, f(2)) = (2, 2 \cdot 4 + 6 \cdot 2) = (2, 20)$ is

$y + 20 = m(2)(x - 2)$ or

$$y + 20 = (4 \cdot 2 + 6)(x - 2)$$
$$y + 20 = 14(x - 2)$$
$$y + 20 = 14x - 28$$
$$y = 14x - 8$$

(11) $f(x) = 2x - \left(\frac{x}{10}\right)^2$ or $f(x) = -\frac{x^2}{100} + 2x$

$m(x) = \left(\frac{-1}{100}\right)(2)x + 2 = \frac{-1}{50}x + 2$ when $x=a$, $m(a) = \frac{-1}{50}x + 2$

At the point $(2, f(2)) = \left(2, -\frac{4}{100} + 2\cdot 2\right) = \left(2, \frac{-1}{25} + 4\right) = \left(2, \frac{99}{25}\right)$

$y - \frac{99}{25} = m(2)(x-2)$ or

$$y - \frac{99}{25} = \left(\frac{-2}{50} + 2\right)(x-2)$$

$$y - \frac{99}{25} = \frac{98}{50}(x-2)$$

$$y - \frac{99}{25} = \frac{98}{50}x - \frac{98}{25}$$

$$y = \frac{98}{50}x + \frac{1}{25}$$

$$y = \frac{49}{25}x + \frac{1}{25}$$

(19) $y = x - \left(\frac{x}{10}\right)^2$ we want all the points where the tangent line is horizontal, ie where the slope is zero.

Here $y = f(x) = -\frac{x^2}{100} + x$. So $m(x) = 2\left(\frac{-1}{100}\right)x - 1 = \frac{-1}{50}x - 1$

we need $m(x) = 0$, so

$$\frac{-1}{50}x - 1 = 0$$

$$\frac{-1}{50}x = 1$$

$$x = -50$$

So the point on the curve where the slope is zero is

$(-50, f(-50)) = \left(-50, -\frac{50^2}{100} + 50\right) = \left(-50, -\frac{50\cdot 50}{100} + 50\right) = (-50, 25)$.

(21) $y = (x+3)(x-5) = x^2 - 2x - 15$

$m(x) = 2(1)x - 2 = 2x - 2$

Slope is zero when $m(x) = 0$, $\quad 2x - 2 = 0$

$$2x = 2$$
$$x = 1$$

Point where slope is zero, $(1, f(1)) = (1, 4(-4)) = (1, -16)$

(28) $y = x^2$. Here $m(x) = 2 \cdot 1x + 0 = 2x$. So at the point x_0, $m(x_0) = 2x_0$. At the point $(x_0, f(x_0)) = (x_0, x_0^2)$ we have the equation of the tangent line is

$y - x_0^2 = m(x_0)(x - x_0)$ or $\quad y - x_0^2 = 2x_0(x - x_0)$

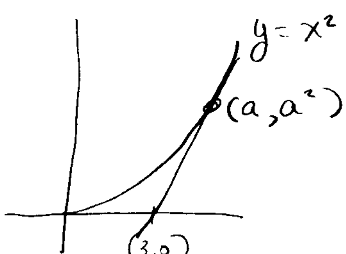
$$y - x_0^2 = 2x_0x - 2x_0^2$$
$$y = 2x_0x - x_0^2.$$

This line intersects the x-axis $(y=0)$ when

$$0 = 2x_0x - x_0^2$$
$$2x_0x = x_0^2$$
$$x = \frac{x_0^2}{2x_0} = \frac{x_0}{2}.$$

Thus the point of intersection is $\left(\frac{x_0}{2}, 0\right)$

(33)

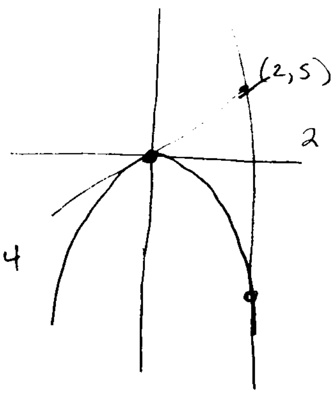


Recall that $m(x) = 2x$, and the tangent line intersects the x-axis at $\left(\frac{x_0}{2}, 0\right)$, or in our case $\left(\frac{a}{2}, 0\right)$. We want this point to be $(3,0)$, so $\frac{a}{2} = 3$, or $a = 6$. Then the equation of the tangent line is $y = 2x_0x - x_0^2$,

or $\quad y = 2(6)x - 6^2 = 12x - 36$.

(34) $y = 4x - x^2$ $\quad (2,5)$

The graph is



Here $m(x) = -1(2)x - 4$
$= -2x - 4$

and the equation of the tangent line at the point $(a, f(a))$
$= (a, 4a - a^2)$ is

$$y - 4a + a^2 = (-2a-4)(x-a)$$
$$y - 4a + a^2 = (-2a-4)x + 2a^2 + 4a$$
$$y = (-2a-4)x + a^2 + 8a$$

we want the tangent line to go through $(2,5)$. Plugging this in,

$$5 = (-2a-4)(2) + a^2 + 8a$$
$$0 = -4a - 8 - 5 + a^2 + 8a$$
$$0 = a^2 + 4a - 13.$$

Using the quadratic formula, $a = \dfrac{-4 \pm \sqrt{16 - 4(-13)}}{2} = \dfrac{-4 \pm \sqrt{16+52}}{2}$

$$= \frac{-4 \pm \sqrt{68}}{2} = \frac{-4 \pm 2\sqrt{17}}{2} = -2 \pm \sqrt{17}.$$

The tangent points are $(2+\sqrt{17}, -29+8\sqrt{17})$, $(-2-\sqrt{17}, -29-8\sqrt{17})$

So the 2 straight lines are

$$y - (-29 + 8\sqrt{17}) = (-2(-2+\sqrt{17}) - 4)(x - (-2+\sqrt{17}))$$
$$y = -2\sqrt{17}\,x + 5 + 4\sqrt{17}$$

and

$$y - (-29 - 8\sqrt{17}) = (-2(-2-\sqrt{17}) - 4)(x - (-2-\sqrt{17}))$$
$$y = 2\sqrt{17}\,x + 5 - 4\sqrt{17}$$