section 2.3 pg 85 # 1, 3, 5, 7, 9, 15, 17, 19, 59, 60

① $\lim_{\theta \to 0} \frac{\theta^2}{\sin\theta} = \lim_{\theta \to 0} \frac{\theta}{\frac{\sin\theta}{\theta}} = \frac{\lim_{\theta \to 0} \theta}{\lim_{\theta \to 0} \frac{\sin\theta}{\theta}} = \frac{0}{1} = 0$

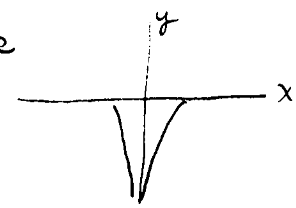
③ $\lim_{\theta \to 0} \frac{1-\cos\theta}{\theta^2} = \lim_{\theta \to 0} \frac{(1-\cos\theta)(1+\cos\theta)}{\theta^2(1+\cos\theta)} = \lim_{\theta \to 0} \frac{1-\cos^2\theta}{\theta^2(1+\cos\theta)}$

$= \lim_{\theta \to 0} \frac{\sin^2\theta}{\theta^2(1+\cos\theta)} = \left[\lim_{\theta \to 0} \left(\frac{\sin\theta}{\theta}\right)^2\right]\left[\lim_{\theta \to 0} \frac{1}{1+\cos\theta}\right] = 1 \cdot \frac{1}{1+1} = \frac{1}{2}$

⑤ $\lim_{t \to 0} \frac{2t}{(\sin t) - t} = \lim_{t \to 0} \frac{2t(\sin t + t)}{(\sin t - t)(\sin t + t)} = \lim_{t \to 0} \frac{2t\sin t + 2t^2}{\sin^2 t - t^2}$

Notice that this didn't help. And we can't just plug in zero

Graphing close to zero it looks like



so $\lim_{t \to 0} \frac{2t}{\sin t - t} = -\infty$

⑦ $\lim_{x \to 0} \frac{\sin 5x}{x} = 5 \lim_{x \to 0} \frac{\sin 5x}{5x}$ Let $\theta = 5x$, then $x \to 0$ implies $\theta \to 0$ too.

$= 5 \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 5$

⑨ $\lim_{x \to 0} \frac{\sin x}{\sqrt{x}} = \lim_{x \to 0} \frac{\sqrt{x}\sin x}{\sqrt{x}\sqrt{x}} = \lim_{x \to 0} \sqrt{x}\frac{\sin x}{x} = \lim_{x \to 0} \sqrt{x} \lim_{x \to 0} \frac{\sin x}{x}$

$= 0 \cdot 1 = 0$

(15) $\lim_{x\to 0} x\sec x\csc x = \lim_{x\to 0} x \cdot \frac{1}{\cos x} \cdot \frac{1}{\sin x} = \lim_{x\to 0} \frac{x}{\sin x} \lim_{x\to 0} \frac{1}{\cos x}$

$= 1 \cdot 1 = 1$

(17) $\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta\sin\theta} = \lim_{\theta\to 0} \frac{(1-\cos\theta)(1+\cos\theta)}{\theta\sin\theta(1+\cos\theta)} = \lim_{\theta\to 0} \frac{1-\cos^2\theta}{\theta\sin\theta(1+\cos\theta)}$

$= \lim_{\theta\to 0} \frac{\sin^2\theta}{\theta\sin\theta(1+\cos\theta)} = \lim_{\theta\to 0} \frac{\sin\theta}{\theta(1+\cos\theta)} = 1 \cdot \frac{1}{1+1} = \frac{1}{2}$

(19) $\lim_{z\to 0} \frac{\tan z}{\sin 2z} = \lim_{z\to 0} \frac{\sin z}{\cos z \sin 2z} = \lim_{z\to 0} \frac{\sin z}{\cos z\,(2\sin z\cos z)}$

$= \lim_{z\to 0} \frac{1}{2\cos^2 z} = \frac{1}{2}$

(59) $f(x) = \frac{x^2-4}{|x-2|}$

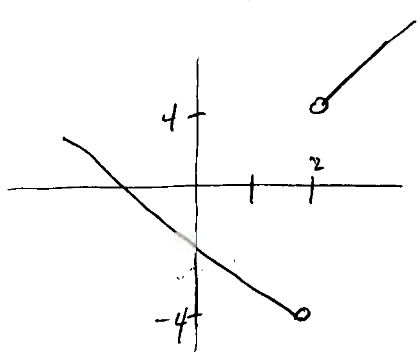
when $x<2$, $|x-2| = -(x-2)$ (makes it positive)

so $\lim_{x\to 2^-} \frac{x^2-4}{|x-2|} = \lim_{x\to 2^-} \frac{(x-2)(x+2)}{-(x-2)} = \lim_{x\to 2^-} -(x+2) = -4$

when $x>2$, $|x-2| = x-2$

so $\lim_{x\to 2^+} \frac{x^2-4}{|x-2|} = \lim_{x\to 2^+} \frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2^+} x+2 = 4$

this means that the limit does not exist.



(60) $f(x) = \dfrac{x^4 - 8x + 16}{|x-2|}$

As in #59, when $x < 2$, $|x-2| = -(x-2)$ and when $x > 2$ $|x-2| = x-2$

$$\lim_{x \to 2^-} \frac{x^4 - 8x + 16}{|x-2|} = \lim_{x \to 2^-} \frac{(x^2-4)^2}{-(x-2)} = \lim_{x \to 2^-} \frac{(x-2)^2(x+2)^2}{-(x-2)} =$$

$$\lim_{x \to 2^-} -(x-2)(x+2)^2 = 0$$

$$\lim_{x \to 2^+} \frac{x^4 - 8x + 16}{|x-2|} = \lim_{x \to 2^+} \frac{(x-2)^2(x+2)^2}{x-2} = \lim_{x \to 2^+} (x-2)(x+2)^2 = 0$$

Since the left and right limits are the same, the limit exists and is equal to 0.

Close to $x = 2$, the graph looks like