section 3.9 pg 195 # 3, 7, 11, 13, 19, 23, 25, 31

(3) $16x^2 + 25y^2 = 400$

$16(2x) + 25(2y)\frac{dy}{dx} = 0$

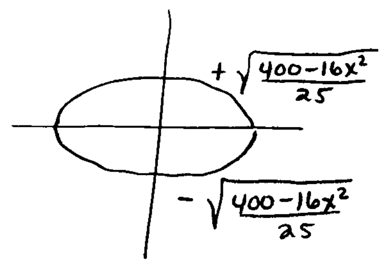
$50y\frac{dy}{dx} = -32x$

$\frac{dy}{dx} = \frac{-32x}{50y} = \frac{-16x}{25y}$

$25y^2 = 400 - 16x^2$

$y^2 = \frac{400-16x^2}{25}$

$y = \pm\sqrt{\frac{400-16x^2}{25}} = \frac{1}{5}\sqrt{400-16x^2}$



$f(x) = \sqrt{\frac{400-16x^2}{25}}$

$f'(x) = \left(\frac{1}{2}\right)\left(\frac{1}{5}\right)\frac{1}{\sqrt{400-16x^2}}(-32x)$

$= \frac{-32x}{10\ y \cdot 5} = \frac{-32x}{50y} = \frac{-16x}{25y}$ so they are the same.

You can also check $g(x) = -\frac{1}{5}\sqrt{400-16x^2}$

(7) $x^{2/3} + y^{2/3} = 1$

$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$

$\frac{2}{3}y^{-1/3}\frac{dy}{dx} = -\frac{2}{3}x^{-1/3}$

$\frac{dy}{dx} = -x^{-1/3}y^{1/3}$

(11) $x\sin y + y\sin x = 1$

$\sin y + x(\cos y)\frac{dy}{dx} + y\cos x + \frac{dy}{dx}\sin x = 0$

$x\cos y\frac{dy}{dx} + \sin x\frac{dy}{dx} = -\sin y - y\cos x$

$\frac{dy}{dx}(x\cos y + \sin x) = -\sin y - y\cos x$

$\frac{dy}{dx} = \frac{-\sin y - y\cos x}{x\cos y + \sin x}$

(13) $2x + 3e^y = e^{x+y}$

$$2 + 3e^y \frac{dy}{dx} = e^{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$2 + 3e^y \frac{dy}{dx} = e^{x+y} + e^{x+y}\frac{dy}{dx}$$

$$2 - e^{x+y} = (e^{x+y} - 3e^y)\, dy/dx$$

$$\frac{dy}{dx} = \frac{2 - e^{x+y}}{e^{x+y} - 3e^y}$$

(19) $xy^2 + x^2y = 2 \qquad P(1,-2)$

$$x(2y)\frac{dy}{dx} + y^2 + 2xy + x^2\frac{dy}{dx} = 0$$

$$1(2)(-2)\frac{dy}{dx} + (-2)^2 + 2(1)(-2) + 1^2\frac{dy}{dx} = 0$$

$$-4\frac{dy}{dx} + 4 - 4 + \frac{dy}{dx} = 0$$

$$-3\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 0$$

equation of tangent line

$$y - (-2) = 0(x-1)$$

$$y + 2 = 0$$

$$y = -2$$

(23) $2e^{-x} + e^y = 3e^{x-y} \qquad P(0,0)$

$$-2e^{-x} + e^y\frac{dy}{dx} = 3e^{x-y}\left(1 - \frac{dy}{dx}\right)$$

$$-2e^{-0} + e^0\frac{dy}{dx} = 3e^0\left(1 - \frac{dy}{dx}\right)$$

$$-2 + \frac{dy}{dx} = 3 - 3\frac{dy}{dx}$$

$$4\frac{dy}{dx} = 5$$

$$\frac{dy}{dx} = 5/4$$

equation of tangent line

$$y - 0 = \frac{5}{4}(x - 0)$$

$$y = \frac{5}{4}x$$

(25) $x^{2/3} + y^{2/3} = 5 \quad P(8,1)$

$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$

$\frac{2}{3} + \frac{2}{3}(8)^{-1/3}\frac{dy}{dx} = 0$

$\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)\frac{dy}{dx} = -2/3$

$\frac{dy}{dx} = \left(-\frac{2}{3}\right)(3) = -2$

equation of tangent line

$y - 1 = -2(x - 8)$

$y - 1 = -2x + 16$

$y = -2x + 17$

(31) $x^2 + y^2 = 4x + 4y$ find points where tangent line is horizontal ie where $\frac{dy}{dx} = 0$

$2x + 2y\frac{dy}{dx} = 4 + 4\frac{dy}{dx}$

$2y\frac{dy}{dx} - 4\frac{dy}{dx} = 4 - 2x$

$\frac{dy}{dx}(2y - 4) = 4 - 2x$

$\frac{dy}{dx} = \frac{4 - 2x}{2y - 4}$

$\frac{4 - 2x}{2y - 4} = 0$

$4 - 2x = 0$

$4 = 2x$

$x = 2$

when $x = 2$,

$2^2 + y^2 = 4(2) + 4y$

$y^2 - 4y - 4 = 0$

$y = \frac{4 \pm \sqrt{16 - 4(-4)}}{2} = \frac{4 \pm \sqrt{32}}{2}$

$= 2 \pm \sqrt{8}$

so tangent line is horizontal at $(2, 2 + \sqrt{8})$ $(2, 2 - \sqrt{8})$