section 2.4

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(53) $x^2 - 5 = 0$ on $[2, 3]$

$f(x) = x^2 - 5$. This is continuous on $[2, 3]$.

$f(2) = 2^2 - 5 = -1$ $f(3) = 3^2 - 5 = 4$

Since $f(2) < 0 < f(3)$, by the intermediate value theorem there is a solution

(55) $x^3 + x + 1 = 0$ on $[-1, 0]$

$f(x) = x^3 + x + 1$ is continuous on $[-1, 0]$

$f(-1) = (-1)^3 + (-1) + 1 = -1$ $f(0) = 0^3 + 0 + 1 = 1$

Since $f(-1) < 0 < f(0)$, by the intermediate value theorem there is a solution

(58) $x^5 - 5x^3 + 3 = 0$ on $[-3, -2]$

$f(x) = x^5 - 5x^3 + 3$ is continuous on $[-3, -2]$.

$f(-3) = (-3)^5 - 5(-3)^3 + 3 = -243 - 5(-81) + 3 = -243 + 405 + 3 = 165$

$f(-2) = (-2)^5 - 5(-2)^3 + 3 = -64 - 5(-8) + 3 = -64 + 40 + 3 = -21$

Since $f(-2) < 0 < f(-3)$, by the intermediate value theorem there is a solution

(59) $x^3 - 4x + 1 = 0$ $f(x) = x^3 - 4x + 1$ is continuous everywhere

$f(-3) = -27 - 4(-3) + 1 = -27 + 12 + 1 = -14$ $f(0) = 1$

$f(-2) = -8 - 4(-2) + 1 = -8 + 8 + 1 = 1$ $f(1) = 1 - 4 + 1 = -4$

$f(-1) = -1 - 4(-1) + 1 = -1 + 5 + 1 = 5$ $f(2) = 2^3 - 4(2) + 1 = 8 - 8 + 1 = 1$

$f(3) = 3^3 - 4 \cdot 3 + 1 = 27 - 12 + 1 = 16$

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$-3$ $-2$ $0$ $2$ $3$

So there are solutions on the intervals $[-3, -2]$, $[-2, 1]$, $[0, 1]$

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(17) $\lim_{x\to -4} \dfrac{\frac{1}{\sqrt{13+x}} - \frac{1}{3}}{x+4} = \lim_{x\to -4} \dfrac{\frac{3-\sqrt{13+x}}{3\sqrt{13+x}}}{x+4} = \lim_{x\to -4} \dfrac{3-\sqrt{13+x}}{3(x+4)\sqrt{13+x}}$

Notice that this is undefined when $x=-4$. But we can try a trick

$= \lim_{x\to -4} \dfrac{(3-\sqrt{13+x})(3+\sqrt{13+x})}{3(x+4)\sqrt{13+x}\,(3+\sqrt{13+x})} = \lim_{x\to -4} \dfrac{9-(13+x)}{(x+4)[9\sqrt{13+x}+3(13+x)]}$

$= \lim_{x\to -4} \dfrac{-(x+4)}{(x+4)[9\sqrt{13+x}+3(13+x)]} = \lim_{x\to -4} \dfrac{-1}{9\sqrt{13+x}+3(13+x)}$

$= \dfrac{-1}{9\sqrt{13-4}+3(13-4)} = \dfrac{-1}{9\sqrt{9}+3\cdot 9} = \dfrac{-1}{27+27} = \dfrac{-1}{54}$

(21) $\lim_{x\to -2^-} \dfrac{x+2}{|x+2|}$ when $x<-2$, $|x+2| = -(x+2)$

$= \lim_{x\to -2^-} \dfrac{x+2}{-(x+2)} = \lim_{x\to -2^-} -1 = -1$

(27) $\lim_{x\to 3^+} \dfrac{x}{x-3}$ Notice that if $x>3$ then $x-3>0$. So

$\lim_{x\to 3^+} \dfrac{x}{x-3} = +\infty$ (instead of $-\infty$)

(29) $\lim_{x\to 1^-} \dfrac{x+1}{(x-1)^3}$ when $x<1$ but close to 1, the numerator is positive and the denominator is negative,

so $\lim_{x\to 1^-} \dfrac{x+1}{(x-1)^3} = -\infty$.

(61) $x^5 + x = 1$ $f(x) = x^5 + x$ is continuous everywhere.

$f(2) = 2^5 + 2 = 64 + 2 = 66$

$f(-2) = (-2)^5 - 2 = -64 - 2 = -66$

since $f(-2) < 1 < f(2)$, by the intermediate value theorem there is a solution on $[-2, 2]$ (there are other intervals that work)

(63) Show that there is a number x between 0 and $\pi/2$ such that $x = \cos x$.

Consider $x - \cos x = 0$, and try to use the intermediate value theorem. $f(x) = x - \cos x$ is continuous everywhere.

$f(0) = 0 - \cos 0 = -1$ $f(\frac{\pi}{2}) = \frac{\pi}{2} - \cos\frac{\pi}{2} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

Since $f(0) < 0 < f(\pi/2)$, there is a solution between 0 and $\frac{\pi}{2}$, ie there is an x between 0 and $\frac{\pi}{2}$ such that $x = \cos x$.