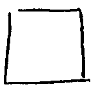
section 3,3 pg 132 # 49, 51, 53, 55, 57, 59, 61

49) we are given that $\frac{dr}{dt} = 2$ in/s. $A = \pi r^2$.

Want: the rate of area increase, ie $\frac{dA}{dt}$

$$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt} = 2\pi r \cdot 2 = 4\pi r.$$

When $r = 10$ in, $\frac{dA}{dt} = 40\pi$ in²/s.

51) 

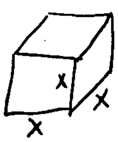
edge increasing at 2 in/s, ie $\frac{dx}{dt} = 2$ in/s $A = x^2$

what rate is area increase? $\frac{dA}{dt}$

$$\frac{dA}{dt} = \frac{dA}{dx} \cdot \frac{dx}{dt} = 2x \cdot 2 = 4x$$

when $x = 10$ in, $\frac{dA}{dt} = 40$ in²/s

Notice that the circle in #49 increases faster than the square.

53) 

edge decreases 2 in/hr, ie $\frac{dx}{dt} = 2$ in/hr

Volume $= x^3$

$$\frac{dV}{dt} = \frac{dV}{dx} \cdot \frac{dx}{dt} = 3x^2 \cdot -2 = -6x^2$$

when $x = 10$, $\frac{dV}{dt} = 600$ in³/hr decreasing

55) $G(t) = f(h(t))$ $h(1) = 4$, $f'(4) = 3$, $h'(1) = -6$. Find $G'(1)$

$$G'(t) = f'(h(t))\, h'(t)$$

$$G'(1) = f'(h(1))\, h'(1) = f'(4) \cdot -6 = 3 \cdot -6 = -18$$

(57)

$\frac{dr}{dt} = 1$ cm/s   want $\frac{dV}{dt}$. $V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt} = 4\pi r^2 \cdot 1 = 4\pi r^2$$

when $r = 10$ cm, $\frac{dv}{dt} = 4\pi \cdot 100 = 400\pi$ cm³/s

(59) air is escaping at a rate of 300π cm³/s.

By the units we can tell that this is $\frac{dV}{dt} = 300\pi$

Here $\frac{dr}{dt} = 3$ cm/s, $V = \frac{4}{3}\pi r^3$ Find r when $\frac{dr}{dt} = 3$

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$$

$$300\pi = 4\pi r^2 \cdot 3$$

$$300\pi = 12\pi r^2$$

$$\frac{300}{12} = r^2$$

$$25 = r^2$$

$r = \pm 5$.   since the radius can't be negative, $r = 5$ cm

(61) decrease in Volume is proportional to surface area. $V = \frac{4}{3}\pi r^3$

ie $\frac{dV}{dt} = k \cdot 4\pi r^2$ where k is a constant. $SA = 4\pi r^2$

when $t=0$, $V = 500$ in³.   when $t = 1$, $V = 250$ in³.

when does the snowball finish melting?

$$\frac{dV}{dt} = k \cdot 4\pi r^2 = \frac{dV}{dr} \cdot \frac{dr}{dt}$$

$$k 4\pi r^2 = 4\pi r^2 \cdot \frac{dr}{dt}$$

so $\frac{dr}{dt} = k$, ie it decreases at a constant rate

so if in 1hr it decreases by 250 in³, by 2hrs it will be gone