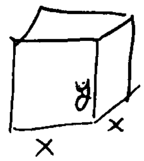
section 3.6 pg 159 # 5, 9, 11, 20, 25

(5)



square base with $x \geq 1$.

No top

total area is 300 in^2.

Find max volume.

$V = x^2 y$

$SA = 300 = x^2 + 4xy$

$300 - x^2 = 4xy$

$y = \dfrac{300 - x^2}{4x}$

$$V = x^2\left(\frac{300 - x^2}{4x}\right) = \frac{(300 - x^2)x}{4} = 75x - \frac{x^3}{4}$$

$V' = 75 - \dfrac{3x^2}{4}$

want to maximize

$75 - \dfrac{3x^2}{4} = 0$

$-\dfrac{3x^2}{4} = -75$

$x^2 = 100$

$x = \pm 10$

can't use -10, so we need to check volume when $x = 1$ and $x = 10$. First find y with these x values

$x = 1$ $\quad y = \dfrac{300 - 1^2}{4 \cdot 1} = 74.75$

$V = x^2 y = 1^2 \cdot 74.75 = 74.75 \text{ in}^3$

$x = 10$ $\quad y = \dfrac{300 - 10^2}{4 \cdot 10} = 5$

$V = 10^2 \cdot 5 = 500 \text{ in}^3$

So max volume is 500 in^3

Technically we need a closed interval to check. Since we can't have $y = 0$, use this to find an endpoint

$$y = \frac{300 - x^2}{4x} = 0$$

$$300 - x^2 = 0$$

$$x^2 = 300$$

$$x = \pm\sqrt{300} \approx 17.32.$$

Note that this corresponds to a box where $y = 0$, So volume is zero. (ie it can't be a max)

(9) x = 1st nonnegative #
y = 2nd nonnegative #   sum is 10

$x + y = 10$ Notice that $0 \le x \le 10$ and $0 \le y \le 10$
(otherwise sum could be more than 10)

Minimize sum of cubes

$$S = x^3 + y^3 = x^3 + (10 - x)^3$$

$$S' = 3x^2 + 3(10-x)^2(-1)$$
$$= 3x^2 - 3(100 - 20x + x^2)$$
$$= 3x^2 - 300 + 60x - 3x^2$$
$$= 60x - 300$$

set to 0
$$60x - 300 = 0$$
$$60x = 300$$
$$x = \frac{300}{60} = 5.$$

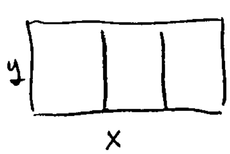
check $x = 10$, $x = 0$, and $x = 5$.

$x=10$, $y=0$
$S = 10^3 + 0^3 = 1000$

$x=0$, $y=10$
$S = 0^3 + 10^3 = 1000$

$x=5$, $y=5$
$S = 5^3 + 5^3 = 250$
minimum

(11)



want to maximize area, $A = xy$
have 600 ft of fence
$$600 = 4y + 2x$$
$$600 - 4y = 2x$$
$$300 - 2y = x$$

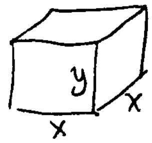
$$A = (300 - 2y)y$$
$$= 300y - 2y^2$$
$$A' = 300 - 4y$$
$$300 - 4y = 0$$
$$300 = 4y$$
$$y = 75$$

degenerate cases are $x=0$ and $y=0$, which gives area $=0$.

$y = 75$, $x = 300 - 2(75) = 150$

$$A = xy = 75 \cdot 150 = 11250 \text{ yd}^2$$

(20)



base cost \$2/ft^2

sides cost \$1/ft^2

can spend \$144 on box

maximize volume

$V = x^2 y$

$V = x^2\left(\frac{144 - 2x^2}{4x}\right)$

$= 36x - \frac{x^3}{2}$

$V' = 36 - \frac{3x^2}{2}$

$36 - \frac{3x^2}{2} = 0$

$36 = \frac{3x^2}{2}$

$24 = x^2$

$x = \pm\sqrt{24} \approx \pm 4.9$

$\text{cost} = 2x^2 + 4xy \cdot 1 = 2x^2 + 4xy$

$144 = 2x^2 + 4xy$

$144 - 2x^2 = 4xy$

$\frac{144 - 2x^2}{4x} = y$

x & y can't be less than 0.

To find the maximum x can be,

set $\frac{144 - 2x^2}{4x} = 0$

$144 - 2x^2 = 0$

$144 = 2x^2$

$x^2 = 72$

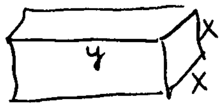
$x = \pm\sqrt{72} \approx \pm 8.5$.

But $x = 8.5$ corresponds to $y = 0$, so no volume.

When $x = \sqrt{24}$, $y = \frac{144 - 2(\sqrt{24})^2}{4\sqrt{24}} = \frac{96}{4\sqrt{24}} = \frac{24}{\sqrt{24}} = \frac{24\sqrt{24}}{24} = \sqrt{24}$

Volume $= x^2 y = (\sqrt{24})^2\sqrt{24} = 24\sqrt{24} = 48\sqrt{6}\ ft^3$ max volume

(25)



Circumference can be a maximum of 100

maximize volume

$V = x^2 y.$

minimum $x = 0$
corresponds to
0 volume

maximum $y + 4x = 100$

$y = 100 - 4x$

So $100 - 4x = 0$

max x can be is 25.

$V = x^2 y = x^2(100 - 4x) = 100x^2 - 4x^3$

$V' = 200x - 12x^2$

$200x - 12x^2 = 0$

$x(200 - 12x) = 0$

$x = 0 \qquad 200 - 12x = 0$

$200 = 12x$

$x = \frac{50}{3}$

when $x = \frac{50}{3}$, $y = 100 - 4x$

$= 100 - 4\left(\frac{50}{3}\right)$

$= \frac{100}{3}$

maximum volume

$V = x^2 y = \left(\frac{50}{3}\right)^2 \cdot \frac{100}{3} = \frac{250000}{3}$

$= 83333\frac{1}{3}\ in^3$