Mathematics 2200 - Quiz Two
Friday, Septmeber 5, 2003

Show all your work and justify each step by citing the appropriate limit law. You may use your note or text book, but **NO** calculator.

1. Evaluate $\lim_{x\to 0} \frac{\sqrt{1+x^2}-\sqrt{1-2x^2}}{x^2}$.

   Solution: This problem is similar to Problem **36** of section **2.2**. I just changed x to x^2 and put into 2 in the second term.

$$\begin{aligned}
\lim_{x\to 0} \frac{\sqrt{1+x^2}-\sqrt{1-2x^2}}{x^2} &= \lim_{x\to 0} \frac{\sqrt{1+x^2}-\sqrt{1-2x^2}}{x^2} \cdot \frac{\sqrt{1+x^2}+\sqrt{1-2x^2}}{\sqrt{1+x^2}+\sqrt{1-2x^2}} \\
&= \lim_{x\to 0} \frac{(1+x^2)-(1-2x^2)}{x^2(\sqrt{1+x^2}+\sqrt{1-2x^2})} \\
&= \lim_{x\to 0} \frac{3x^2}{x^2(\sqrt{1+x^2}+\sqrt{1-2x^2})} \\
&= \lim_{x\to 0} \frac{3}{\sqrt{1+x^2}+\sqrt{1-2x^2}} \\
&= \frac{3}{\lim_{x\to 0}\sqrt{1+x^2}+\lim_{x\to 0}\sqrt{1-2x^2}} \quad \text{(quotient and sum laws)} \\
&= \frac{3}{2} \quad \text{(substitution law)}
\end{aligned}$$

□

2. Evaluate $\lim_{x\to 0} \frac{1}{x}\left(\frac{1}{\sqrt{16+x}}-\frac{1}{4}\right)$

   Solution: This is the Problem 34 of section **2.2**(I changed 9 to 16 and 3 to 4)

$$\begin{aligned}
\lim_{x\to 0} \frac{1}{x}\left(\frac{1}{\sqrt{16+x}}-\frac{1}{4}\right) &= \lim_{x\to 0} \frac{1}{x}\frac{4-\sqrt{16+x}}{4\sqrt{16+x}} \\
&= \lim_{x\to 0} \frac{4-\sqrt{16+x}}{4x\sqrt{16+x}} \cdot \frac{4+\sqrt{16+x}}{4+\sqrt{16+x}} \\
&= \lim_{x\to 0} \frac{16-(16+x)}{4x\sqrt{16+x}(4+\sqrt{16+x})} \\
&= \lim_{x\to 0} \frac{-x}{4x\sqrt{16+x}(4+\sqrt{16+x})} = \lim_{x\to 0} \frac{-1}{4\sqrt{16+x}(4+\sqrt{16+x})} \\
&= -\frac{1}{128} \quad \text{quotient, product and substitution laws.}
\end{aligned}$$

□

3. Find the slop-predictor function $m(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ for $f(x) = x^2 + \frac{2}{x}$. Then find the equation of the tangent line at $(1, 3)$.

Solution: We have done a similar problem in class. First we compute $f(x) - f(a)$ and factorize the factor $(x - a)$ from it.

$$\begin{aligned} f(x) - f(a) &= \left(x^2 + \frac{2}{x}\right) - \left(a^2 + \frac{2}{a}\right) \\ &= (x - a)(x + a) + \left(\frac{2}{x} - \frac{2}{a}\right) \\ &= (x - a)(x + a) + 2\frac{(a - x)}{xa} = (x - a)(x + a - \frac{2}{xa}) \end{aligned}$$

This implies $\frac{f(x) - f(a)}{x - a} = x + a - \frac{2}{xa}$. Hence

$$m(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a} \left(x + a - \frac{2}{xa}\right) = 2a - \frac{2}{a^2}.$$

When $a = 1$, $f(1) = 3$ and $m(1) = 0$. Hence the equation of the tangent line is

$$y - 3 = 0 \cdot (x - 2), \quad y = 3.$$

□