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**Mathematics 2200 - Quiz Five**  
**Wednesday, Oct. 23, 2002**

1. Find all points on the curve  $y = \frac{1}{2 \sin^2 x + 3 \cos^2 x}$  where the tangent line is horizontal.

*Solution:* This problem is directly taken from the home work and we have went through the problem in the class.

$$\frac{dy}{dx} = -\frac{4 \sin x \cos x - 6 \cos x \sin x}{(3 \sin^2 x + 2 \cos^2 x)^2} = \frac{2 \sin x \cos x}{(3 \sin^2 x + 2 \cos^2 x)^2}.$$

To find the points on the curve where the tangent line is horizontal, we set  $\frac{dy}{dx} = 0$ . This implies that  $2 \sin x \cos x = 0$  and  $\sin x = 0$  or  $\cos x = 0$ . Hence,  $x = k\pi$  or  $k\pi + \pi/2$  for  $k$  integers. When  $x = k\pi$ ,  $y = 1/2$  and when  $x = k\pi + \pi/2$ ,  $y = 1/3$ . Therefore the tangent lines are horizontal at points  $(k\pi, \frac{1}{3})$  and  $(k\pi + \frac{\pi}{2}, \frac{1}{2})$ .  $\square$

2. Using the definition of the derivative  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  to find the derivative of the function  $y = \cos x$

*Solution:* We have found the derivative of  $y = \sin x$  in the class. You just need to change sin to cos in that proof. This is not a homework problem, but I told that I will put it in this quiz and I have to keep my promise!

Let  $f(x) = \cos x$ . Since  $\cos(x+h) = \cos x \cos h - \sin x \sin h$ ,

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cos x(\cos h - 1) - \sin x \sin h}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cos x(\cos h - 1)}{h} - \lim_{h \rightarrow 0} \frac{\sin x \sin h}{h} \\ &= \cos x \lim_{h \rightarrow 0} \frac{(\cos h - 1)}{h} - \sin x \lim_{h \rightarrow 0} \frac{\sin h}{h} \\ &= -\sin x \end{aligned}$$

Here we have used the result from Chapter 2:

$$\lim_{h \rightarrow 0} \frac{\sin h}{h} = 1 \quad \text{and} \quad \lim_{h \rightarrow 0} \frac{\cos h - 1}{h} = 0.$$

$\square$

3. A water trough is to be made from a long strip of tin 3 ft wide by bending up at an angle  $\theta$  a 1-ft on each side. What angle  $\theta$  would maximize the cross-sectional area?

*Solution:* This is also from the home work (section 3.7). The cross-section is a trapezoid with height  $h = \sin \theta$  and lower base  $a = 1$ , up base  $b = 1 + 2 \cos \theta$ . So the area

$$S = \frac{1}{2} \sin \theta (2 + 2 \cos \theta) = \sin \theta + \sin \theta \cos \theta = \sin \theta + \frac{1}{2} \sin 2\theta$$

And the domain of this function is  $0 \leq \theta \leq 2\pi/3$  (Why?). We next find the derivative

$$\frac{dS}{d\theta} = \cos \theta + \cos 2\theta = \cos \theta + (2 \cos^2 \theta - 1) = 2 \cos^2 \theta + \cos \theta - 1.$$

Next we want to find the critical point by setting  $\frac{dS}{d\theta} = 0$  and obtain

$$2 \cos^2 \theta + \cos \theta - 1 = 0 \quad (2 \cos \theta - 1)(\cos \theta + 1) = 0$$

Hence we get  $\cos \theta = -1$  or  $\cos \theta = 1/2$ . But  $\cos \theta = -1$  is not in the domain. Hence  $\cos \theta = 1/2$  implies  $\theta = \pi/3$ . we find the value of  $S$  at  $\theta = 0$ ,  $\theta = 2\pi/3$  and  $\theta = \pi/3$ :

$$S(0) = 0, \quad S(2\pi/3) = \frac{\sqrt{3}}{4} \quad \text{and} \quad S(\pi/3) = \frac{3\sqrt{3}}{4}$$

Hence  $\max S = \frac{3\sqrt{3}}{4}$ .

□