

**Mathematics 2210 - Quiz Three**  
**Monday, September 27, 2004**

Show all your work. You may use your note or text book.

1. Sketch the region bounded by the given curves, then find its area.

a)  $x = y^2$ ,  $x = 8 - 2y$ .

*SOLUTION:* First find the intersection points of two curves by solving  $x = y^2$   $x = 8 - 2y$  to get  $y^2 = 8 - 2y$ , and  $y = 2$  and  $y = -4$ . Then the area

$$A = \int_{-4}^2 (8 - 2y - y^2) dy = \left( 8y - y^2 - \frac{1}{3}y^3 \right) \Big|_{-4}^2 = 36.$$

□

b)  $y = x^2$ ,  $y = 4(x - 1)^2$ .

*SOLUTION:* Find the intersection points first by solving

$$x^2 = 4(x - 1)^2 \quad \pm x = 2(x - 1), \quad x = 2 \quad \text{and} \quad x = \frac{2}{3}.$$

The area is

$$A = \int_{2/3}^2 [x^2 - 4(x - 1)^2] dx = \left. \frac{x^3}{3} - \frac{4(x - 1)^3}{3} \right|_{2/3}^2 = \frac{4}{3} - \frac{12}{81} = \frac{32}{27}.$$

□

2. Evaluate the limit  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{m_i}{m_i^2 + 9} \Delta x$  by computing the value of the appropriate related integral, where  $m_i$  is the midpoint of the  $i$ -th subinterval  $[x_{i-1}, x_i]$  of a partition of the interval  $[0, 6]$  into  $n$  subintervals each of length  $\Delta x$ .

*SOLUTION:* The limit can be written as the integral

$$\int_0^6 \frac{x}{x^2 + 9} dx = \left. \frac{1}{2} \ln(x^2 + 9) \right|_0^6 = \frac{1}{2} \ln 45 - \frac{1}{2} \ln 9 = \frac{1}{2} \ln 5.$$

□