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Mathematics 2200 - Quiz Four
Wednesday, October 5, 2005

Find the maximum and minimum values attained by the following functions on the indicated closed intervals.

1. $f(x) = |2x + 1| + |3x - 2|$; $[-1, 1]$.

Solution: This problem is similar to the homework problem. We can rewrite the function with the absolute sign as follows:

$$f(x) = \begin{cases} -(2x + 1) - (3x - 2) = -5x + 1 & -1 \leq x \leq -\frac{1}{2} \\ (2x + 1) - (3x - 2) = -x + 3 & -\frac{1}{2} < x \leq \frac{2}{3} \\ (2x + 1) + (3x - 2) = 5x - 1 & \frac{2}{3} < x \leq 1 \end{cases}$$

This implies

$$f'(x) = \begin{cases} -5 & -1 < x < -\frac{1}{2} \\ -1 & -\frac{1}{2} < x < \frac{2}{3} \\ 5 & \frac{2}{3} < x < 1 \end{cases}$$

This function is continuous in $[-1, 1]$ and differentiable inside except at points $x = -1/2$ and $x = 2/3$. They are the critical points of the function. Find the values of the function at the critical points and boundary points, we have

$$f(-1) = 6, \quad f(-\frac{1}{2}) = 3.5, \quad f(\frac{2}{3}) = \frac{7}{3} \quad \text{and} \quad f(1) = 4.$$

Hence the maximum value of $f(x)$ over the interval is 6 and the minimum value is $\frac{7}{3}$. □

2. $y = 3x^{\frac{1}{2}} - x^{\frac{3}{2}}$; $[0, 4]$

Solution: First note that the function is continuous over the interval $[0, 4]$. Find the derivative first

$$f'(x) = \frac{3}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = \frac{3 - 3x}{2x^{\frac{1}{2}}}.$$

Hence the critical points are $x = 1$ and $x = 0$ since $f'(1) = 0$ and $f'(0)$ does not exist.

Next we find the values of the function at the critical points and boundary points:

$$f(0) = 0, \quad f(1) = 2, \quad \text{and} \quad f(4) = -2.$$

Hence the maximum value is 2 and the minimum value is -2 . □

3. Express the derivative $\frac{dy}{dx}$ in terms of x : $y = u(1 - u)^3$ and $u = \frac{1}{x^4}$.

Solution: By the chain rule, we have $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$. We find $\frac{dy}{du}$ and $\frac{du}{dx}$ first

$$\frac{dy}{du} = (1 - u)^3 + u \cdot 3(1 - u)^2(-1) = (1 - u)^2(1 - 4u) \quad \frac{dy}{dx} = -4x^{-5}$$

Hence

$$\frac{dy}{dx} = (1 - x^{-4})^2(1 - 4x^{-4})(-4)x^{-5}.$$

□

4. Express the derivative $\frac{dy}{dx}$ in terms of x : $y = \frac{u^2}{u^2 + 1}$ and $u = \frac{x^2}{x^2 + 1}$

Solution: Note that the two functions are the same.

$$\frac{dy}{du} = \frac{2u(u^2 + 1) - u^2 \cdot 2u}{(u^2 + 1)^2} = \frac{2u}{(u^2 + 1)^2}$$

and

$$\frac{du}{dx} = \frac{2x}{(x^2 + 1)^2}$$

This yields

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{2u}{(u^2 + 1)^2} \cdot \frac{2x}{(x^2 + 1)^2} = \frac{2x^2}{(x^2 + 1)} \cdot \frac{1}{\left(\frac{x^4}{(x^2 + 1)^2} + 1\right)^2} \cdot \frac{2x}{(x^2 + 1)^2}.$$

We can simplify it as follows:

$$\frac{dy}{dx} = \frac{4x^3(x^2 + 1)}{(2x^4 + 2x^2 + 1)^2}.$$

□