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Mathematics 2200 - Quiz Five
Friday, Oct. 21, 2005

1. Find all points on the curve $y = \dfrac{1}{2\sin^2 x + 3\cos^2 x}$ where the tangent line is horizontal.

Solution: This problem is directly taken from the home work and we have went through the problem in the class.

$$\frac{dy}{dx} = -\frac{4\sin x\cos x - 6\cos x\sin x}{(3\sin^2 x + 2\cos^2 x)^2} = \frac{2\sin x\cos x}{(3\sin^2 x + 2\cos^2 x)^2}.$$

To find the points on the curve where the tangent line is horizontal, we set $\dfrac{dy}{dx} = 0$. This implies that $2\sin x\cos x = 0$ and $\sin x = 0$ or $\cos x = 0$. Hence, $x = k\pi$ or $k\pi + \pi/2$ for k integers. When $x = k\pi$, $y = 1/2$ and when $x = k\pi + \pi/2$, $y = 1/3$. Therefore the tangent lines are horizontal at points $(k\pi, \frac{1}{3})$ and $(k\pi + \frac{\pi}{2}, \frac{1}{2})$. □

2. Find the derivative of the function $y = x^{\ln x}$ and $y = x^{\sin x}$.

Solution: We need to use the method of logarithmic differentiation. For the first function $y = x^{\ln x}$, we have $lny = \ln x^{\ln x}$. Apply the property of the logarithmic function, we have $\ln y = \ln x \cdot \ln x = (\ln x)^2$. This implies

$$\frac{1}{y}\cdot\frac{dy}{dx} = 2\ln x\cdot\frac{1}{x}, \quad \text{hence} \quad \frac{dy}{dx} = 2x^{\ln x}\cdot\frac{\ln x}{x}.$$

The second function $y = x^{\sin x}$ implies $\ln y = \sin x\ln x$. Differentiate with respect to x, we have

$$\frac{1}{y}\cdot\frac{dy}{dx} = \cos x\ln x + \sin x\cdot\frac{1}{x}, \quad \text{hence} \quad \frac{dy}{dx} = x^{\sin x}\left(\cos x\ln x + \frac{\sin x}{x}\right).$$

□

3. A water trough is to be made from a long strip of tin 3 ft wide by bending up at an angle θ a 1-ft on each side. What angle θ would maximize the cross-sectional area?

Solution: This is also from the home work (section 3.7). The cross-section is a trapezoid with height $h = \sin\theta$ and lower base $a = 1$, up base $b = 1 + 2\cos\theta$. So the area

$$S = \frac{1}{2}\sin\theta(2 + 2\cos\theta) = \sin\theta + \sin\theta\cos\theta = \sin\theta + \frac{1}{2}\sin 2\theta$$

And the domain of this function is $0 \leq \theta \leq 2\pi/3$(Why?). We next find the derivative

$$\frac{dS}{d\theta} = \cos\theta + \cos 2\theta = \cos\theta + (2\cos^2\theta - 1) = 2\cos^2\theta + \cos\theta - 1.$$

Next we want to find the critical point by setting $\dfrac{dS}{d\theta} = 0$ and obtain

$$2\cos^2\theta + \cos\theta - 1 = 0 \quad (2\cos\theta - 1)(\cos\theta + 1) = 0$$

Hence we get $\cos\theta = -1$ or $\cos\theta = 1/2$. But $\cos\theta = -1$ is not in the domain. Hence $\cos\theta = 1/2$ implies $\theta = \pi/3$. we find the value of S at $\theta = 0$, $\theta = 2\pi/3$ and $\theta = \pi/3$:

$$S(0) = 0, \quad S(2\pi/3) = \frac{\sqrt{3}}{4} \quad \text{and} \quad S(\pi/3) = \frac{3\sqrt{3}}{4}$$

Hence $\max S = \frac{3\sqrt{3}}{4}$. □