PRINT NAME: ____________________

SIGNATURE: ____________________

Student ID#: ____________________

Mathematics 2200 - Final Examinations

Friday, August 3, 2001

Problem #	Points	Score
PART A	60	
B1	20	
B2	10	
B3	10	
B4	10	
B5	10	
Bonus	10	
Total	130	

This Exam has two parts:

A: 15 multiple choice questions (60 marks).
Indicate your answer to each multiple-choice question in **PART A** by checking the approciate answer. No Partial credit will be given in this part.

B: 6 questions. Show all your work or credit will not be given.

NOTE:

1. Before you start, check that the Exam has **12 pages**. There is **NO** blank pages. You can use the back of the pages for rough work.
2. **NO** aid of any kind is allowed. **NO CALCULATORS**.
3. **DO NOT TEAR OUT THIS PAGE OR ANY OTHER PAGE**.
4. **DO NOT TALK TO EACH OTHER**.

!!! GOOD LUCK !!!

PART A

Each of the following 15 multiple-choice questions has **exactly one** correct answer. 4 marks for a correct answer, 0 for no answer or a worng answer or an unclear answer or indicating more than one answer. You are not required to justify your answer in this part.

1. $\lim_{x\to 2} \dfrac{x^2+3x-10}{x^2+x-6} =$

   (a) 0

   (b) $\frac{4}{5}$

   (c) $\frac{2}{3}$

   (d) $\frac{7}{5}$

   (e) Undefined

2. $\lim_{x\to 0} \dfrac{(x+1)^3-3x-1}{2x^2} =$

   (a) $\frac{3}{2}$

   (b) $\frac{1}{2}$

   (c) 0

   (d) $-\frac{1}{2}$

   (e) Undefined

3. $\lim_{x\to 0} \left(\dfrac{\sin 3x}{2x} + \dfrac{\sin 7x}{\sin 4x} + \dfrac{1-\cos x}{x^2} \right) =$

   (a) $\frac{13}{4}$

   (b) $\frac{11}{4}$

   (c) $\frac{15}{4}$

   (d) $\frac{11}{2}$

   (e) $\frac{7}{4}$

4. $\lim_{x\to\infty}(x - \sqrt{x^2 - 5x}) =$

(a) 0

(b) ∞

(c) 1

(d) $\frac{5}{2}$

(e) $\frac{5}{4}$

5. $\lim_{x\to\infty}\dfrac{\sqrt[3]{8x^3 - 8x + 1}}{3x - 4} =$

(a) ∞

(b) $\frac{2}{3}$

(c) $\frac{1}{3}$

(d) $-\frac{1}{4}$

(e) $-\frac{2}{3}$

6. Let $f(x) = x + \sqrt{x}$ on $[1, 4]$. The Mean Value Theorem says that there must be some number c between 1 and 4 so that $f'(c) =$ the average slope of $f(x)$ on $[1, 4]$. The number c must be

(a) $\frac{5}{2}$

(b) 3

(c) 2

(d) $\frac{11}{4}$

(e) $\frac{9}{4}$

7. If $f(x) = \sqrt{\dfrac{7}{2} + \cos^2 x}$, then $f'(\frac{\pi}{4}) =$

(a) $-\frac{1}{2}$

(b) $\frac{1}{2}$

(c) $\frac{1}{4}$

(d) $\sqrt{2}$

(e) $-\frac{1}{4}$

8. Let $f(x) = 1 + x^3$. If $g(x) = f^{-1}(x)$, i.e., if g is the inverse of f, then $g'(9) =$

(a) $\frac{1}{12}$

(b) $\frac{1}{243}$

(c) $\frac{1}{8}$

(d) $\frac{1}{24}$

(e) $\frac{1}{128}$

9. The graph of $f(x) = 4x^{\frac{1}{3}} - x^{\frac{4}{3}}$ $(x \geq 0)$ has a horizontal tangent line at $x =$

(a) 4

(b) $\sqrt{3}$

(c) $\frac{1}{3}$

(d) 1

(e) 0

10. Let $f(x) = \dfrac{\ln x}{x^2}$, for $x > 0$. Then f has

(a) a point of inflection at $x = \sqrt{e}$

(b) a global max. at $x = \frac{1}{\sqrt{e}}$

(c) a global min. at $x = \frac{1}{\sqrt{e}}$

(d) a global max. at $x = \sqrt{e}$

(e) a global min. at $x = \sqrt{e}$

11. Let $f(x) = xe^{-\frac{x^2}{2}}$. Then the graph of f is concave upward in the interval

(a) $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$

(b) $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$

(c) $(-\sqrt{3}, \sqrt{3})$

(d) $(1, \infty)$

(e) $(-\infty, -1)$

12. The function $f(x) = x - \dfrac{1}{x}$ has

(a) a local min. at $x = 1$ and $x = -1$.

(b) a local max. at $x = 1$ and $x = -1$.

(c) a local min. at $x = 1$ and a local max. at $x = -1$.

(d) a local max. at $x = 1$ and a local min. at $x = -1$.

(e) no local max. and no a local min.

13. The function $f(x) = x\left(\frac{2x-1}{x-1}\right)^2$ has a slant asymptote

(a) the line $y = 4x - 2$.

(b) the line $y = 4x + 4$.

(c) the line $y = 4x$.

(d) the line $y = 4x - 4$.

(e) the line $y = 4x + 2$.

14. If $\sin(x + 2y) = 2x\cos y$, then the value of $\frac{dy}{dx}$ at the point $(0, \pi)$ must be

(a) $-\frac{3}{2}$

(b) -1

(c) -2

(d) 2

(e) 0

15. The product of two positive numbers is 12, what is the smallest possible value of the sum of their squares?

(a) 25

(b) 24

(c) 40

(d) $18\sqrt{2}$

(e) 18

PART B

Answer all questions in PART B in spaces provided. Show all your work. Any answer in PART B without justification may receive very little or no credit. Use the back of each page for rough work.

1.[20 points] Use any suitable method to find $\frac{dy}{dx}$ for each of the following function. There is no need to simplify your final answer for this question.

(a). $y = \ln\left(\frac{x^2 - 1}{x^2 + 1}\right)$.

(b). $y = \sin^2(\cos^2 x)$.

(c). $y = e^{-x^2}(2x^2 - x)$.

(d). $y = \frac{2x^3 - x^2 + 2x + 1}{x^2}$.

(e). $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$.

2.[10 points] Apply the definition of derivative to find the derivative of $f(x) = \dfrac{1+x^2}{x}$, and also find the equations of the lines which are tangent to f and with slope equal to -3.

3.[10 points]A man is at point A on a bank of straight river, 3km wide, and wants to reach point B, 8 km downstream on the opposite bank, as quickly as possible. He could row his boad directly across the river to point C and then run to B, or he could row directly to B, or he could row to some point D between C and B and then then run to B If he can row at 8 km/h and run at 10km/h. Where should he land to reach B as soon as possible?

4.[10 points] We must make a cylindrical can with volume 125 in^3 by cuting its top and bottom from squares of metal and forming its curved side by bending a rectangular sheet of metal to match its ends. What radius r and height h of the can will minimize the total amount of material required for the rectangle and two squares? Justify your conclusion by citing the theorem you have used.

5.[10 point] P is a point on the positive x-axis. Q is a point on the positive y-axis and O is the origin. What is the smallest possible area of triangle OPQ if the line passing through P and Q is required to be tangent to the curve $y = 3 - x^2$. Justify your conclusion by citing the theorem you have used.

(**Hint**: Let PQ tangent to the curve $y = 3 - x^2$ at $(a, 3 - a^2)$. Express $|OP|$, $|OQ|$ and the area of OPQ in terms of a).

Bonus [10 points] Let a be a fixed constant. Evaluate the following limit $\lim_{x\to 0} \dfrac{\sin(a+2x) - 2\sin(a+x) + \sin a}{x^2}$. [**Hint**: $\sin\alpha + \sin\beta = 2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$]