

PRINT NAME: _____

Student ID#: _____

Mathematics 2200 - Test Four
Wednesday, Dec. 5, 2001

Problem #	Points	Score
1	20	
2	20	
3	20	
4	20	
bonus	10	
Total	90	

Show all work or credit will not be given.

No aids are allowed. **NO Calculators!**

Do Not Tear Out Any Pages.

1. Find the vertical, horizontal asymptotes and the slant asymptotes of the given functions when they exist:

(a) $f(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$.

(b) $f(x) = \frac{2x^2 - x - 1}{x^2 - 1}$.

2. Find the critical points and inflection points of the function $g(x) = (x + 1)^2 e^{-x}$, and apply the first or second derivative test to find the local maxima and minima.

3. A cylindrical can is to be made to hold 1 L. of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can. We take 1 L of oil to 1 m^3 . You must quote the appropriate theorem.

4. State the mean value theorem and show that the function $f(x) = x + \frac{1}{x}$ satisfies the hypotheses of the mean value theorem on the interval $[1, 5]$. Find the number c in the interval $[1, 5]$ that satisfies the conclusion of the mean value theorem.

5. * **Bonus question** (Note: This is not an easy question and will be marked very strictly) P is a point on the positive x -axis. Q is a point on the positive y -axis and O is the origin. What is the smallest possible area of triangle OPQ if the line passing through P and Q is required to be tangent to the curve $y = 3 - x^2$. Justify your conclusion by citing the theorem you have used.
(**Hint:** Let PQ be tangent to the curve $y = 3 - x^2$ at $(a, 3 - a^2)$. Express $|OP|$, $|OQ|$ and the area of OPQ in terms of a).