MAT 4000/6000 ASSIGNMENT #1

DUE Wed. Jan. 16, 2008.

All problems for this assignment are practice problems. They are for the most part matters we have covered in class, and are meant as a review for the test.

Practice Problems: **Don't hand these problems in.**

I. **Let G be a group and let x, g be elements of G, and let H be a subgroup of G. Prove that $C_G(gxg^{-1}) = gC_G(x)g^{-1}$ and $N_G(gHg^{-1}) = gN_G(H)g^{-1}$.**

II. **Let G be a group and let H be a subgroup of G. Let S be the set of left cosets of H in G. Let $\phi : G \to Perm(S)$ be defined by $\phi(g)(aH) = gaH$ (i.e ϕ is the homomorphism giving the action of G on the left cosets by translation.) Prove that the kernel of ϕ is $\bigcap aHa^{-1}$, the intersection over all a in G. Conclude that this intersection is a normal subgroup of G and $G/\bigcap aHa^{-1}$ is isomorphic to a subgroup of S_n (symmetric group), where n is the number of elements in S.**

III. **Prove that there is no simple group of order 48.**

IV. **Let $G = Gl_2(3)$, and let $x = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Let H be the normalizer of $< x >$ in G. Describe H and $\bigcap aHa^{-1}$, the intersection over all a in G. Use this to prove that $PGL_2(3)$ is isomorphic to S_4.**

V. **A Russian woman claims to have clairvoyant powers. She was shown pictures of 7 different people with 7 different illnesses, and asked to match each person with his or her illness. She matched 5 correctly. What is the probability that she would get exactly this number correctly if she were guessing randomly?**