TITLES AND ABSTRACTS

Plenary Talks

Ciprian Demeter, Institute of Advanced Study

Modulation invariant bilinear $T(1)$ theorem

We prove a $T(1)$ theorem for bilinear singular integral operators (trilinear forms) with a one-dimensional modulation symmetry. This is joint work with Á. Bényi, A.R. Nahmod, C.M. Thiele, R.H. Torres and P. Villarroya

Burak Erdogan, University of Illinois at Urbana-Champaign

Dispersive estimates for the Schrödinger equations

Nikos Frantzikinakis, University of Memphis

New examples of sets of multiple recurrence

A classical result of Szemeredi (1975) states that every subset of the integers with positive density contains configurations of the form $x, x+n, ..., x+kn$. Several refinements of this result seek to restrict the scope of n. For example, Bergelson and Leibman (1996) using ergodic theory showed that n can be a perfect square. We will survey some new recent refinements of Szemeredi's theorem and discuss some open problems.

Dmitri Jakobson, McGill University

Estimates from below for the spectral function and for the remainder in Weyl's law on negatively-curved surfaces.

We obtain asymptotic lower bounds for the spectral function of the Laplacian on compact manifolds. In the negatively curved case, thermodynamic formalism for hyperbolic flows is applied to improve the estimates. Our results can be considered pointwise versions (on a general manifold) of Hardy's lower bounds for the error term in the Gauss circle problem. We next discuss lower bounds for the remainder in Weyl's law on negatively-curved surfaces. Our approach works in variable negative curvature, and is based on wave trace asymptotics for long times, equidistribution of closed geodesics and small-scale microlocalization. This is joint work with I. Polterovich and J. Toth.

Izabella Laba, University of British Columbia

Arithmetic progressions in Salem sets

We address the following question: if $E \subset [0,1]$ is a set of Hausdorff dimension α, must E contain a 3-term arithmetic progression? We construct a counterexample of dimension $\alpha = 1$. On the other hand, we have a positive result when E is a Salem set (i.e. E supports a measure whose Fourier transform has appropriate decay at infinity). This is joint work with Malabika Pramanik.

Daniel Oberlin, Florida State University

Model surfaces for a convolution problem

This talk concerns certain k-surfaces in $\mathbb{R}^d$ which support measures satisfying optimal or nearly optimal convolution estimates.

Malabika Pramanik, University of British Columbia

Maximal averages over monomial polyhedra

We derive L^p bounds for maximal functions associated to families of polyhedra in $\mathbb{R}^n$ defined by a large number of monomial inequalities. The problem is motivated by applications in several complex variables, and its solution uses techniques from convex geometry and optimization. This is joint work with Alexander Nagel.

Keith Rogers, Universidad Autonoma de Madrid

Local smoothing and pointwise convergence for the Schrödinger equation

We will show that the Schrödinger operator $e^{it\Delta}$ is bounded from $L^q_\alpha(\mathbb{R}^n)$ to $L^q(\mathbb{R}^n \times [0,1])$ for all $\alpha > 2n(1/2 - 1/q) - 2/q$ and $q \geq 2 + 4/(n+1)$. This is almost sharp with respect to the Sobolev index.

We will also show that the Schrödinger maximal operator $\sup_{0<t<1} |e^{it\Delta} f|$ is bounded from $H^s(\mathbb{R}^n)$ to $L^2_{\mathrm{loc}}(\mathbb{R}^n)$ when $s > s_0$ if and only if it is bounded from $H^s(\mathbb{R}^n)$ to $L^2(\mathbb{R}^n)$ when $s > 2s_0$. A corollary is that $\sup_{0<t<1} |e^{it\Delta} f|$ is bounded from $H^s(\mathbb{R}^2)$ to $L^2(\mathbb{R}^2)$ when $s > 3/4$.

Stephen Wainger, University of Wisconsin at Madison

An expository introduction to the circle method of Hardy, Littlewood and Ramanujan.

Mate Wierdl, University of Memphis

Independence of powers and related problems

In 1912, Hardy and Littlewood proved the following result: for any irrational number b, the set

$$\{(b, b), (2b, 2^2 b), (3b, 3^2 b), \dots\} \tag{1}$$

is dense mod 1 in the two dimensional torus. One can interpret this result as saying that the sets

$$\{1b, 2b, 3b, \dots\}$$

and

$$\{1^2 b, 2^2 b, 3^2 b, \dots\}$$

are independent. The corresponding result of Weyl on the uniform distribution of the sequence in (1) is a more precise expression of this independence.

In our talk, we will discuss several recent results which seem to give further (hopefully) interesting illustration for this independence. Our examples are partly from number theory (bases, intersective sets) and partly from ergodic theory (recurrence, convergence). We will mention some (sometimes loosely) related unsolved problems as well.

Joint work with Frantzikanikis, Johnson, Lesigne.

James Wright, University of Edinburgh

Van der Corput's lemma mod N

A standard tool to estimate oscillatory integrals in harmonic analysis is the well-known lemma due to van der Corput which gives a sharp decay bound for one dimensional oscillatory integrals when some derivative of the phase is bounded below. We prove an analogous result for complete exponential sums and as a consequence we obtain a new proof of a classical result due to Hua.

Invited Short Talks

Michael Bateman, Indiana University

Directional Maximal Operators in the Plane

We completely characterize the boundedness of planar directional maximal operators on L^p. More precisely, if Ω is a set of directions, we show that M_Ω, the maximal operator associated to line segments in the directions Ω, is unbounded on L^p, for all $p < \infty$, precisely when Ω admits Kakeya-type sets. In fact, we show that if Ω does not admit Kakeya sets, then Ω is a generalized lacunary set, and hence M_Ω is bounded on L^p, for $p > 1$.

Neal Bez, University of Birmingham

Near L^1 mapping properties of maximal operators along curves

For certain curves Γ in $\mathbb{R}^d$ and increasing convex functions $\Phi : [0, \infty) \to [0, \infty)$, we address the question of whether the corresponding maximal operator along Γ is of weak type $\Phi(L)$.

Spyros Dendrinos, University of Bristol

Fourier restriction on curves and a geometric inequality

We present a Fourier restriction result for general polynomial curves in $\mathbb{R}^n$. Measuring the Fourier restriction with respect to the affine arclength measure of the curve, we obtain a universal bound for the class of all polynomial curves of bounded degree. Our method relies on establishing a geometric inequality for general polynomial curves which is of interest in its own right.

Derrick Hart, University of Missouri

Sums and products in finite fields

Let A be subset of a finite field. In recent years there has been considerable interest in sums versus products problems, i.e. the fact that either the product set or the sum set of A is large. We use estimates on the analog of the discrete Radon transform in finite fields to yield a few results of this flavor.

Doowon Koh, University of Missouri

Extension theorems for the Fourier transform associated with non-degenerate quadratic surfaces in vector spaces over finite fields.

We study the restriction of the Fourier transform to quadratic surfaces in vector spaces over finite fields. In two dimensions, we obtain the sharp result by considering the sums of arbitrary two elements in the subset of quadratic surfaces on two dimensional vector spaces over finite fields. For higher dimensions,we estimate the decay of the Fourier transform of the characteristic functions on quadratic surfaces so that we obtain the Tomas-Stein exponent. Using incidence theorems, we also study the extension theorems in the restricted settings to sizes of sets in quadratic surfaces. Estimates for Gauss and Kloosterman sums and their variants play an important role.

Richard Oberlin, University of California at Los Angeles

The (d, k) Kakeya Problem

A (d, k) set is a subset of $\mathbb{R}^d$ containing a translate of every k-dimensional plane. The (d, k) Kakeya problem is to determine the minimum size of a (d, k) set.

Ioannis Parissis, Georgia Institute of Technology

A sharp bound for the Stein-Wainger oscillatory integral

Let $\mathcal{P}_d$ denote the space of all real polynomials of degree at most d. It is an old result of Stein and Wainger that

$$\sup_{P\in\mathcal{P}_d}\left|p.v.\int_{\mathbb{R}} e^{iP(t)}\frac{dt}{t}\right| \le C_d$$

for some constant C_d depending only on d. On the other hand, Carbery, Wainger and Wright claim that the true order of magnitude of the above principal value integral is $\log d$. We prove that

$$\sup_{P\in\mathcal{P}_d}\left|p.v.\int_{\mathbb{R}} e^{iP(t)}\frac{dt}{t}\right| \sim \log d.$$

Matthew Smith, University of Georgia

On solution-free sets for simultaneous additive equations

I will use a combination of the Hardy-Littlewood circle method and the methods developed by Gowers in his recent proof of Szemerédi's Theorem on long arithmetic progressions to obtain quantitative estimates for the upper density of a set containing no solutions to a translation and dilation invariant system of diagonal polynomials of degrees $1, 2, \dots, k$.

Armen Vagharshakyan, Georgia Institute of Technology

Small Ball Inequality in All Dimensions

Let h_R denote an L^∞ normalized Haar function adapted to a dyadic rectangle $R \subset [0,1]^d$. We show that for any choices of coefficients $\alpha(R)$, we have the following lower bound on the L^∞ norms of the sums of such functions, where the sum is over rectangles of a fixed volume:

$$n^{\frac{d-1}{2}-\eta}\Big\|\sum_{|R|=2^{-n}}\alpha(R)h_R(x)\Big\|_{L^\infty([0,1]^d)} \gtrsim 2^{-n}\sum_{|R|=2^{-n}}|\alpha(R)|\,, \quad \text{for some} \quad 0<\eta<\frac{1}{2}\,.$$

The point of interest is the dependence upon the logarithm of the volume of the rectangles. With $n^{(d-1)/2}$ on the left above, the inequality is trivial, while it is conjectured that the inequality holds with $n^{(d-2)/2}$.

This result is known in the case of $d = 2$. The value of η we obtain will be even better than in a recent paper of the authors. There is a corresponding lower bound on the L^∞ norm of the Discrepancy function of an arbitary distribution of a finite number of points in the unit cube in three dimensions.

Joint work with Dmitry Bilyk and Michael Lacey.

Stefan Valdimarsson, University of California at Los Angeles

The Optimisers for the Brascamp–Lieb inequality

By studying the proof of the Brascamp–Lieb inequality via the heat flow method we determine all optimisers for the inequality.

Matthew Wright, University of Missouri

Mixed Boundary Value Problems for the Stokes System

This talk will focus on proving the well-posedness of the mixed boundary value problem for the Stokes system in a class of Lipschitz domains. Using the method of layer potentials, we first reduce the problem to solving a boundary integral equation. Since we lack a boundary Korn inequality for solutions fo the Stokes system, we are forced to derive a new Rellich-type estimate in order to show that the boundary integral operator is semi-Fredholm. Using recent results by Russell Brown and Irina Mitrea concerning the mixed boundary problem for the Lame system, we are then able to utilize a deformation argument to conclude that the operator in question is an isomorphism.