Math 3100 - Practice Exam 1

Problem 1. Let the sequence $\{a_n\}$ be given recursively by, $a_1 = 0$, and $a_{n+1} = a_n + 3n(n+1)$. Use induction to show that $a_n = n(n^2 - 1)$.

Problem 2.

(a) Carefully state the definition of convergence of a sequence $\{a_n\}$ to real number L.

(b) Using the definition of convergence, prove that

$$\lim_{n\to\infty} \frac{4n+5}{1-2n} = -2$$

Problem 3. Let $\{a_n\}$ be a sequence, and assume that there is a real number $0 < r < 1$ such that

$$0 < a_{n+1} < r\, a_n \quad \text{for all} \quad n$$

Prove that $\lim n \to \infty\ a_n = 0$.

(Hint: Use induction to estimate a_n by the n-th term of a geometric sequence.)

Problem 4. Determine which of the following statements are true. If you think one is true, prove it! If you think it is false, construct a counter example!

(a) If $\lim_{n\to\infty} a_n = \infty$ and $\lim_{n\to\infty} b_n = 0$ then the sequence $\{a_n b_n\}$ is bounded.

(b)* If $\lim_{n\to\infty} a_n = \infty$ and $\lim_{n\to\infty} b_n = L \neq 0$ then the sequence $\{a_n b_n\}$ is unbounded.

Problem 5. Let $\{a_n\}$ be an increasing sequence and assume that $\lim_{n\to\infty} = L$. Prove that $a_n \leq L$ for all n.