# Math 3100 - Practice Test II.

**Problem 1.** For each of the following series state whether it converges absolutely, converges conditionally or diverges. Justify your answers!

(a) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$

(b) $\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$

(c) $\sum_{n=1}^{\infty} \sin(1/n^2)$

(d) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^{3/2}}$

Hint: Prove, and use the fact that: $\lim_{n\to\infty} \frac{\ln(n)}{n^p} = 0$ for every $p > 0$)

(e) $\sum_{n=1}^{\infty} \frac{10^n}{\sqrt{n!}}$

(f) $\sum_{n=1}^{\infty} \frac{n^{100}}{\sqrt{2^n}}$

**Problem 2.** Prove the following statements.

(a) If $a_n$ is bounded and $\sum_{n=1}^{\infty} b_n$ is absolute convergent, then $\sum_{n=1}^{\infty} a_n b_n$ is also absolute convergent.

(b) If $\{a_n > 0\}$, $\{b_n > 0\}$, and $\lim_{n\to\infty} \frac{a_n}{b_n} = 1$, then

$$\sum_{n=1}^{\infty} a_n \quad \text{convergent} \quad \equiv \quad \sum_{n=1}^{\infty} b_n \quad \text{convergent}$$

(c) If $\{a_n > 0\}$, $\{b_n > 0\}$, $\{a_n\}$ is bounded and $\sum_{n=1}^{\infty} b_n$ is convergent, then $\sum_{n=1}^{\infty} a_n b_n$ is convergent.

(d) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n^2$ converges, as well.

**Problem 3.** Provide counterexamples to the following false statements.

(a) If $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=1}^{\infty} a_n$ converges.

(b) If $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = 1$, then $\sum_{n=1}^{\infty} a_n$ diverges.

(c) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} |a_n|$ converges.

**Problem 4.** For what values of $x$ do the following sequences converge?

(a) $\sum_{n=1}^{\infty} \frac{(2x)^n}{2n+1}$

(b) $\sum_{n=1}^{\infty} \frac{(x-1)^n\, n}{2^n}$

**Problem 5.** Evaluate the following series

(a) $\sum_{n=1}^{\infty} \frac{1}{5^n}$

(b) $\sum_{n=3}^{\infty} \frac{2}{5^n}$

Solution keys, coming soon :)