Homework 7, Due Monday, October 20, 2003

1. Let $\vec{e}_1 = \langle 1, 0\rangle$, $\vec{e}_2 = \langle 0, 1\rangle$, $\vec{v} = \langle 2, 3\rangle$, $\vec{w} = \langle -11, 4\rangle$, $\vec{z} = \langle a, b\rangle$.

Compute the following dot products:

i) $e_1 \cdot \vec{z}$ ii) $e_2 \cdot \vec{z}$ iii) $\vec{v} \cdot \vec{z}$ iv) $\vec{v} \cdot \vec{w}$ v) $\vec{v} \cdot \vec{v}$

2. Compute the following matrix products.

i) $\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 7 \end{bmatrix}$

ii) $\begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix}$

iii) $\begin{bmatrix} \frac{2}{3} & -3 \\ -2 & 9 \end{bmatrix} \begin{bmatrix} 9 & 12 \\ 2 & \frac{8}{3} \end{bmatrix}$

3. Assume an orthonormal frame $\mathcal{F}$ has been chosen. Let $\vec{v} = \langle 2, 3\rangle_{\mathcal{F}}$. Find the $\mathcal{F}$-components of the vector resulting from each of the following operations.

i) Rotate $\vec{v}$ by angle $\pi/2$.

ii) Rotate $\vec{v}$ by angle $-\pi/2$.

iii) Rotate angle $\vec{v}$ by the acute angle whose cosine is $5/13$.

The next two exercises are designed to demonstrate that one can prove facts about trigonometry by matrix methods. In problem 4, you will prove the addition formulas, and in problem 5 you will prove the law of cosines.

4. Given two angles α and β, the rotation through angle $\alpha + \beta$ can be accomplished in two steps – first rotate through α then through β.

i) Express the previous sentence as an equation in which a matrix is written as the product of two matrices.

ii) Prove the addition formulas for sine and cosine by carrying out the matrix multiplication you see in part i.

5. i) Draw a nonzero vector. Label it $\vec{v}$. Place the base of a second nonzero vector at the tip of $\vec{v}$. Label the second vector $\vec{w}$. Draw the sum of the two vectors by the "dance instruction" method, and label it $\vec{v} + \vec{w}$. Let θ denote the angle formed by $\vec{v}$ and $\vec{v} + \vec{w}$.

ii) **Without introducing components,** express $|\vec{v} + \vec{w}|^2$ as a dot product. Expand this dot product by the distributive law. (I repeat: Don't introduce components!).

iii) From your formula in part ii, prove the law of cosines. You will need the trigonometric formula for the dot product proved in class.