Assignment 3, Due Monday, January 28:

Section 9: 2 – 3

3. Let $B : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^k$ be a bilinear function. (Familiar and important examples include the cross product, the dot product and matrix multiplication.) Prove that B is differentiable, and find its derivative. (In this problem it is more convenient to view the derivative as a linear transformation than as a matrix.)

4. (Product Rule) Let $B : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^k$ be a bilinear function. Let $\mathcal{U} \subset \mathbb{R}^l$ be an open subset and let $f : \mathcal{U} \to \mathbb{R}^n$ and $g : \mathcal{U} \to \mathbb{R}^m$ be differentiable functions. Let $h : \mathcal{U} \to \mathbb{R}^k$ be defined by

$$h(a) = B(f(a), g(a)) \ .$$

i) Find the derivative of h. (Again, think of the derivative as a linear transformation.)
ii) Recover the familiar product rule of one-variable calculus as a special case.

5. Let $\mathcal{U} \subset \mathbb{R}^m$ be an open set, and consider a function $f : \mathcal{U} \to \mathbb{R}^n$. Prove that the following statements are equivalent:

a. For every open set $\mathcal{W} \subset \mathbb{R}^m$, $f^{-1}(\mathcal{W})$ is open.
b. f satisfies the ϵ-δ definition of continuity.

6. Provide the bootstrapping argument mentioned in class to get from the inverse function theorem for C^1 functions to the inverse function theorem for C^k functions.