Assignment 5, Due Wednesday, February 13:

Section 14: 8

Section 15: 2, 3

4. Let T and S be rectifiable subsets of $\mathbb{R}^n$, with $T \subset S$. Let $f : S \to \mathbb{R}$ be a function such that $\int_S f$ exists. Prove that $\int_T f$ exists.

5. Let $\mu = (\mu_1, \mu_2) : \mathbb{R}^3 \to \mathbb{R}^2$ and $f : \mathbb{R}^2 \to \mathbb{R}$ be smooth functions. Assume there is a rectangle $Q \subset \mathbb{R}^2$, such that for all $t \in [0,1]$, $f \circ \mu(x,y,t) = 0$ for $(x,y) \notin Q$. For $t \in [0,1]$, let

$$I(t) = \int_{(x,y)\in\mathbb{R}^2} f(\mu_1(x,y,t), \mu_2(x,y,t)) det \frac{\partial(\mu_1, \mu_2)}{\partial(x,y)} \ .$$

Prove that $I(t)$ is constant.