

ABSTRACT ALGEBRA: A Geometric Approach

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Errata and Typographical Errors

p. 17, lines 3-4. N is the smallest positive integer having \dots . Note, in particular, that N cannot be prime; now, suppose \dots

p. 18, Exercise 3. $\gcd(0, m) = |m|$.

p. 18, Exercise 13. $\mu = |ab|$ and $|ab|/d$.

p. 21, line 4. Delete “)”.

p. 24, line 8. Delete the second “ p .”

p. 35, Exercise 38. Using $e = 101$, check that $d = 317$. In the first line of the table, 1663 should be 1633. \dots Using the same N and $e = 1601$, \dots

p. 39, line 14. $\dots a^{-1}(ab) = a^{-1} \cdot 0 = 0 \dots$

p. 47, line 10. Given $n \in \mathbb{Z}$, exactly one of the statements

$$n \in \mathbb{Z}^+, n = 0, \text{ or } -n \in \mathbb{Z}^+$$

is true. (This is called the trichotomy principle.)

line 15. \mathbb{Z}^+ should be \mathbb{Z}^+ .

line 21. Given $x \in F$, exactly one of the statements

$$x \in F^+, x = 0, \text{ or } -x \in F^+$$

is true.

p. 66, Exercise 14. This is correct when $v \geq 0$. What's the answer when $v < 0$?

p. 66, Exercise 17. Delete the first dx .

p. 91, lines 1-4. $p_1(x), \dots, p_m(x)$ should be *distinct* irreducible polynomials. And $\deg(r_j(x)) < \deg(p_j(x)^{v_j})$ for $j = 1, \dots, m$.

p. 92, Exercise 9. Parts a. and b. are much harder than I thought when I wrote the problem. Even deciding if there are any irreducible elements of $\mathbb{Z}_6[x]$ seems quite difficult.

p. 116, line 3. (ya) should be (ya^{-1}) at the end of the line.

p. 117, Example 3, last line. $\langle 2 \rangle = \langle 4 \rangle \subset \mathbb{Z}_6$, and $4 = 2 \cdot 2$, so 4 is a multiple of 2 by a zero-divisor. On the other hand, $4 = -2$, so 4 is *also* a multiple of 2 by a unit! In fact, in \mathbb{Z}_n it can be shown that $\langle a \rangle = \langle b \rangle \iff a = sb$ for some unit s . (**N.B.** One can construct a commutative ring R having elements a, b satisfying $\langle a \rangle = \langle b \rangle$ and yet $a = sb$ for *no* unit s !)

p. 118, line -10. Proposition 2.2 of Chapter 3.

pp. 123-4, Exercise 16. A few symbols didn't print here:

a. Given an ideal $J \subset S$, define

$$\mathcal{J} = \{a \in R : \phi(a) \in J\} \subset R.$$

(This is usually denoted by $\phi^{-1}(J)$.) Prove that \mathcal{J} is an ideal.

b. Given an ideal $I \subset R$, define

$$\mathcal{J} = \{\phi(a) : a \in I\} \subset S.$$

\dots Prove that \mathcal{J} is an ideal, provided ϕ maps onto S . \dots

- p. 128, last line: $\bar{3}$ should be 3.
- p. 130, line 3 of the proof of Theorem 2.4. ev_α maps $F[x]$ onto $F[\alpha]$ and has
- p. 133, **Remark** and Corollary 2.8. In the displayed statement marked with (*), we must assume that $\alpha \notin \mathbb{Q}$. In the application, we should point out that $\alpha \notin \mathbb{Q}$ (e.g., by applying the result of Exercise 2.2.11a).
- p. 138, Exercise 28a. We must assume $p \neq 0$.
- p. 143, line 8. $\psi(a + bi) = a + 3b \pmod{5}$.
- p. 163, line 4. for some nonnegative integer r ...
- p. 184, lines 9–10. ... in \mathbb{Z}_5^\times there is only one.
- p. 194, lines 3–4. That is, the conjugate of $F = F_1$ (which is the flip fixing vertex 1) by R is F_2 (which is the flip fixing vertex 2).
- p. 258, line 8 of Proof of Theorem 5.8. $k(aH) \subset aH$.
- p. 260, Exercise 18. Assume also that if $\alpha = 1$, then $m > 1$.
- p. 265, last line "... extending ϕ (...) and carrying α to α' ."
- p. 266, line 10. "If $\tilde{\phi}(p(\alpha)) = 0, \dots$ "
 Proof of Proposition 6.4. Assume $K \neq F$.
- p. 268, line 4. $F'[\alpha_j]$ should be $F'[\alpha'_j]$.
- p. 269, line 8. "splits in K ."
- p. 272, line 15. $-\omega$ should be $\bar{\omega}$.
- p. 275, line 8. t should be ℓ .
- p. 277, Exercise 5. $\phi\psi^2$ should be $\psi\phi^2$.
- p. 302, line 1. $\{\infty\}$ should be \mathbb{P}_∞^1 .
- p. 308, Figure 11. α, β, γ should be $\beta, \gamma,$ and $\alpha,$ respectively.
- p. 316, Exercise 18. This is false with the definitions provided. Indeed, we should really say that *six points are in general position provided no three are collinear and they do not all lie on a conic*. Then Exercise 18 becomes a triviality.
- p. 319, line 6. $\mathbf{x}_1, \dots, \mathbf{x}_5 \in \mathbb{R}^3$.
- p. 324, line 4. $\mathbf{v}_2 = \frac{1}{3\sqrt{2}}(1, -4, 1)$.
- p. 324, line -5. $\mathcal{Y}_i = \sqrt{|\lambda_i|}x_i$.
- p. 325, line 11. $z_i = \sqrt{|\lambda_i|}y_i$.
- p. 342, Exercise 20. A few symbols didn't print here:
 ... Associate to the line \overline{PQ} the point $\mathcal{A} = [a_{01}, a_{02}, a_{03}, a_{12}, a_{13}, a_{23}] \in \mathbb{P}^5$.
- Prove that \mathcal{A} is a well-defined point in \mathbb{P}^5 determined only by the line. (... the point $\mathcal{A} \in \mathbb{P}^5$ doesn't change. ... $a'_{ij} = p'_i q'_j - p'_j q'_i$, we have $\mathcal{A}' = \mathcal{A} \in \mathbb{P}^5$.)
 - Show that \mathcal{A} satisfies the (homogeneous) quadratic equation ...
 - Given two lines ℓ and m , let \mathcal{A} and \mathcal{B} be the corresponding points in \mathbb{P}^5
- p. 361, line 15. "two" should be "four."
- p. 363, Exercise 6. The reference should be to Lemma 5.7.
- p. 367, Exercise 30b,c. The factors of $\frac{1}{2}$ should be 2's.

p. 371, line 6. Q should be \mathbb{Q} .

p. 378, Exercise 5. $h = g \circ f$; interchange f and g in parts a., b., c.

p. 387, Figure 1. The labels P and P^{-1} should be interchanged.

p. 401, Exercise 1a. ... prove λ^n is an eigenvalue of T^n

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