Spring, 2008 | T. Shifrin

MATH 4250/6250
PROBLEM SET #8

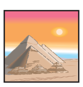
DUE April 15, 2008.

Instructions: Turn in (at least) five problems of your choice, but **including any underlined problem(s)**. Graduate students should include at least one "pyramid" problem.

Problems to work but not hand in: §3.1: #1a, 3.

 §3.1: #1bc, 4, 5, 7.

 §3.1: #**<u>9</u>** (or 3.3.9a), 10 (or 3.3.9b), 12, 13 (or 3.3.10).

 §3.1: #14, 15.

A. What happens when we try to apply the Gauss-Bonnet Theorem to the simplest non-smooth surface, a right circular cone? Let R denote the surface given by

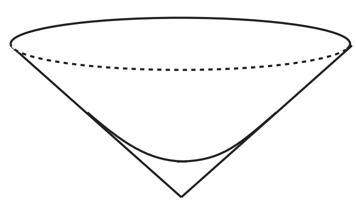
$$\mathbf{x}(u,v) = (u\tan\phi\cos v, u\tan\phi\sin v, u)\,, \quad 0 < u \le u_0,\ 0 \le v \le 2\pi\,,$$

and ∂R its boundary curve.

(1) Show that if we make R by gluing the edges of a circular sector ("pacman") of central angle β, as indicated in Figure 2.4.6, then $\displaystyle\int_{\partial R} \kappa_g\, ds = 2\pi\sin\phi = \beta$. We call β the *cone angle* of R at its vertex.

(2) Show that Theorem 3.1.5 holds for R if we add $2\pi - \beta$ to $\iint_R K\, dA$.

(3) Show that we obtain the same result by "smoothing" the cone point, as pictured below. (Hint: Interpret $\iint_R K\, dA$ as the area of the image of the Gauss map.)



Remark. It is not hard to give an explicit $\mathcal{C}^2$ such smoothing. For example, construct a $\mathcal{C}^2$ convex function f on $[0,1]$ with $f(0) = f'(0) = 0$, $f(1) = f'(1) = 1$, and $f''(1) = 0$.