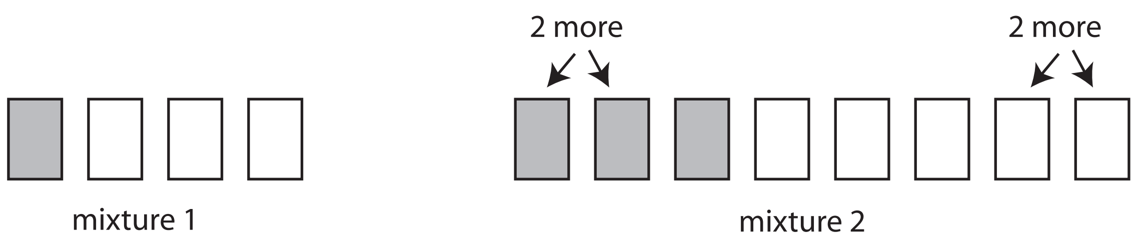
0.1 Ratio and Proportion

Class Activity 0A: Comparing Mixtures

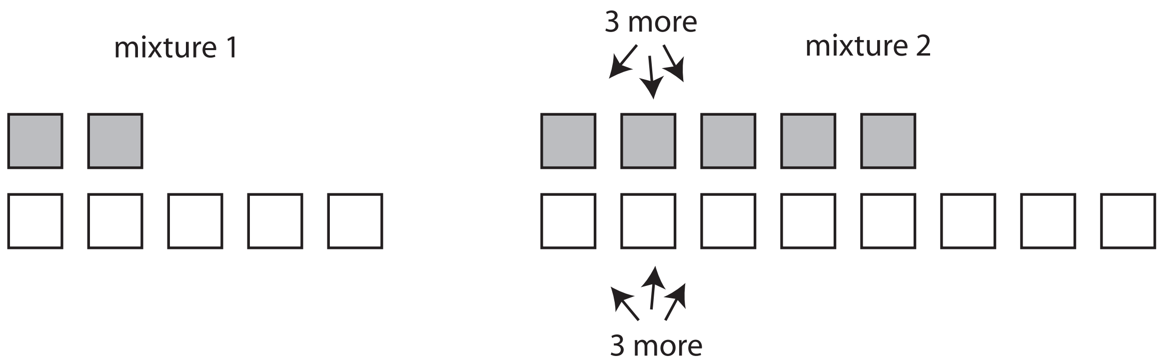
1. There are two containers, each containing a mixture of 1 cup red punch and 3 cups lemon lime soda. The first container is left as it is, but somebody adds 2 cups red punch and 2 cups lemon lime soda to the second container.



Will the two punch mixtures taste the same? Why or why not? Try to develop several different ways to determine if the two mixtures should taste the same or not.

If the mixtures are available, try them to see if they taste the same or not.

2. There are two containers, each containing a mixture of 2 drops blue paint and 5 drops yellow paint (all drops are the same size). The first container is left as it is, but somebody adds 3 drops blue paint and 3 drops yellow paint to the second container.

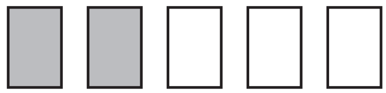


Will the two paint mixtures be the same shade of green? Why or why not? Try to develop several different ways to determine if the two mixtures should look the same or not.

If possible, make the paint mixtures to see if they look the same or not.

Class Activity 0B: Using Ratio Tables

One batch of a certain shade of green paint is made by mixing 2 pails of blue paint with 3 pails of yellow paint.



1. Fill in the next ratio table about batches of the paint mixture described above.

# of batches	1	2	3	4	5	6	7	8	9	10
# pails blue paint	2									
# pails yellow paint	3									
# pails green paint produced	5									

2. Describe the patterns in the 2nd 3rd and 4th rows as you go to the right in the ratio table. Explain why there are those patterns.

3. Describe how the entries in the 6th column of the table are related to the entries on the 1st column of the table.

   Describe how entries in the 8th column of the table are related to the entries in the 1st column of the table.

4. If the table were to continue in the same pattern, how would the entries in the 100th column be related to the entries in the 1st column?

   How would the entries in the nth column of the table be related to entries in the 1st column of the table? Explain your reasoning.

5. Find relationships among the entries in a column. In particular, describe multiplicative relationships (relationships that use multiplication) among the entries in a column.

6. Use your findings from parts 4 and 5 to determine in several different ways how to fill in the next ratio table which is based on the same paint mixture as above.

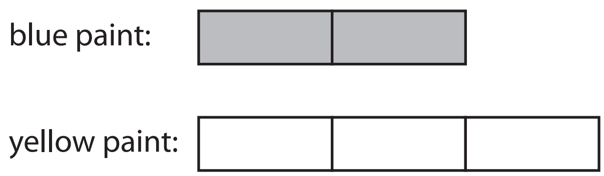
# of batches	1						
# pails blue paint	2	100			22		
# pails yellow paint	3		45			75	
# pails green paint produced	5			1000			85

7. Make ratio tables for the two punch mixtures of Class Activity 0A and use your ratio tables to compare the drink mixtures. Will the mixtures taste the same? If not, which mixture will taste more like red punch?

Class Activity 0C: Using Singapore Strip Diagrams to Solve Ratio Problems

The mathematics textbooks used in elementary school in Singapore (see [?]) show how to solve ratio problems by using simple strip diagrams like the one on this page.

Suppose a certain shade of green paint is made by mixing blue paint with yellow paint in a ratio of 2 to 3.



For each of the problems on this page, use the same shade of green paint as above, which is made by mixing blue paint with yellow paint in a ratio of 2 to 3. Explain how to solve the problems by using the strip diagram.

1. If you will use 40 pails of blue paint, how many pails of yellow paint will you need?

2. If you will use 48 pails of yellow paint, how many pails of blue paint will you need?

3. If you want to make 100 pails of green paint, how many pails of blue paint and how many pails of yellow paint will you need?

4. At lunch, there was a choice of pizza or a hot dog. Three times as many students chose pizza as chose hot dogs. All together, 160 students got lunch. How many students got pizza and how many got a hot dog? Draw a strip diagram to help you solve this problem. Explain your reasoning.

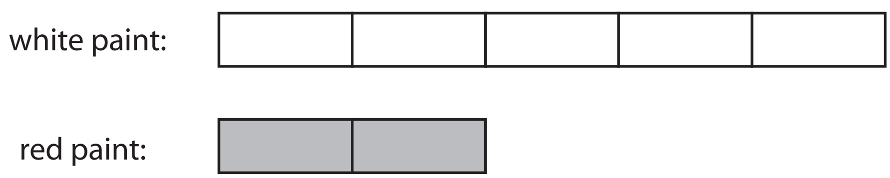
5. The ratio of Shauntay's cards to Jessica's cards is 5 to 3. After Shauntay gives Jessica 15 cards, both girls have the same number of cards. How many cards do Shauntay and Jessica each have now? Draw a strip diagram to help you solve this problem. Explain your reasoning.

6. The ratio of Shauntay's cards to Jessica's cards is 5 to 2. After Shauntay gives Jessica 12 cards, both girls have the same number of cards. How many cards do Shauntay and Jessica each have now? Draw a strip diagram to help you solve this problem. Explain your reasoning.

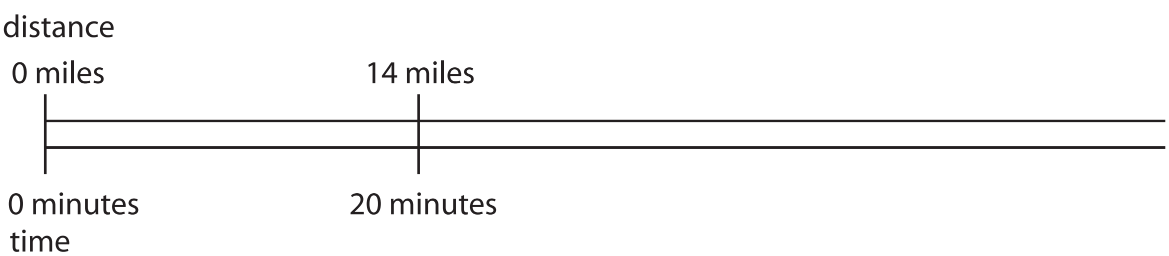
7. Make a new problem for your students by modifying problem 5 or 6. Change the ratio and change the number of cards that Shauntay gives to Jessica. When you make these changes, which ratios will make the problem easier, and which ratios will make it harder? Once you have chosen a ratio, can the number of cards that Shauntay gives to Jessica be any number, or do you need to take care in choosing this number? Explain.

Class Activity 0D: Using Simple Reasoning to Find Equivalent Ratios and Rates

1. For a certain shade of pink paint, the ratio of white paint to red paint is 5 to 2. Explain how to use the picture below to find at least 6 different quantities of white and red paint that will be in the ratio 5 to 2 (and so will make the same shade of pink). Include at least 3 mixtures where the quantities of paint are not both whole numbers of units (you may choose any units you like, such as gallons, liters, pails, or something else).



2. Driving at a constant speed, you drove 14 miles in 20 minutes. On the "double number line" below, show different distances and times that would give you the same speed. Use simple, logical reasoning to determine these equivalent rates.



3. Could you use a double number line (as in part 2) to show equivalent paint ratios for part 1?

   What are some advantages and some limitations of double number lines?

   Could you use a strip diagram (as in part 1) to show speeds for part 2?

4. A box of Brand A laundry detergent washes 20 loads of laundry and costs \$6. A box of Brand B laundry detergent washes 15 loads of laundry and costs \$5.

   (a) In the ratio tables that follow, fill in equivalent rates of loads washed per dollar. Include some examples where the number of loads washed is less than 15 and the cost is less than \$5. Explain your reasoning.

Brand A								
loads washed	20							
cost	\$6							

Brand B								
loads washed	15							
cost	\$5							

   If possible, use your tables to make a statement comparing the two brands of laundry detergent.

(b) Explain how to use simple logical reasoning to fill in the next tables with equivalent rates. Then use the tables to make statements comparing the two brands of laundry detergent.

Brand A		
loads washed	20	
cost	\$6	\$1

Brand B		
loads washed	15	
cost	\$5	\$1

Brand A		
loads washed	20	1
cost	\$6	

Brand B		
loads washed	15	1
cost	\$5	

(c) Robert says that Brand B is less expensive because it costs \$5 but Brand A costs \$6. Discuss Robert's reasoning. What would Robert benefit from learning about?

5. Traveling at a constant speed, a scooter went $\frac{3}{4}$ of a mile in 4 minutes. Use simple, logical reasoning to help you determine the answers to the next questions.

   (a) How far did the scooter go in the following amounts of time:

   8 minutes? 12 minutes? 10 minutes?

   2 minutes? 1 minute?

   (b) How long did it take the scooter to go 1 mile?

   Which aids will be helpful in answering the above questions: a strip diagram (as in part 1), a double number line (as in part 2), or a table (as in part 3)?

Class Activity 0E: Solving Proportions With Multiplication and Division

1. If you mix fruit juice and bubbly water in a ratio of 3 to 5 to make a punch, then how many liters of fruit juice and how many liters of bubbly water will you need to make 24 liters of punch?

   (a) Explain how to use a strip diagram to solve this problem.

   (b) Suppose you will give this paint problem to students but you decide to change 24 liters to 10 liters. Will the problem be just as easy to solve or not? How can you change the 24 liters so that the problem is still easy to solve with a strip diagram? How can you change the 24 liters so that the problem becomes harder to solve with a strip diagram?

2. If you mix fruit juice and bubbly water in a ratio of 3 to 5 to make a punch, then how many liters of fruit juice and how many liters of bubbly water will you need to make 10 liters of punch?

   (a) Use multiplication, division, and logical reasoning to explain how to fill in the blanks in the following table of equivalent ratios, thereby solving this punch problem:

# liters juice	3		
# liters bubbly water	5		
# liters punch	8	1	10

   (b) Describe the strategy for solving the punch problem that the table in part (a) helps you use. What is the idea behind the way the table was created?

3. If you mix $\frac{3}{4}$ cup of red paint with $\frac{2}{3}$ cup of yellow paint to make an orange paint, then how many cups of red paint and how many cups of yellow paint will you need if you want to make 15 cups of the same shade of orange paint?

   (a) Use multiplication, division, and logical reasoning to explain how to fill in the blanks in the following table with equivalent rations, thereby solving this paint problem.

# cups red	$\frac{3}{4}$			
# cups yellow	$\frac{2}{3}$			
# cups orange		17	1	15

   (b) Describe the strategy for solving the paint problem that the table in part (a) helps you use. What is the idea behind the way the table was created?

4. Chandra made a milkshake by mixing $\frac{1}{2}$ cup of ice cream with $\frac{3}{4}$ cup of milk. Use the most elementary reasoning you can to determine how many cups of ice cream and milk Chandra should use if she wants to make the same milkshake (i.e., using the same ratios) for the following amounts:

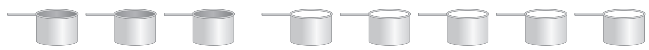
   (a) using 3 cups of ice cream;

   (b) to make 3 cups of milkshake.

5. Russell was supposed to mix 3 tablespoons of weed killer concentrate with $1\frac{3}{4}$ cups of water to make a weed killer. By accident, Russell put in an extra tablespoon of weed killer concentrate, mixing 4 tablespoons of weed killer concentrate with $1\frac{3}{4}$ cups of water. How much water should Russell add to his mixture so that the ratio of weed killer concentrate to water will be the same as in the correct mixture? Use the most elementary reasoning you can to solve this problem.

Class Activity 0F: Ratios, Fractions, and Division

For a certain shade of orange paint, the ratio of red to yellow is 3 to 5. For each of the following fractions and division problems, write a question about the orange paint that will be answered by the given fraction (viewed as parts of a whole) or by solving the associated division problem.



1. $\frac{3}{8}$ or $3 \div 8$

2. $\frac{5}{8}$ or $5 \div 8$

3. $\frac{3}{5}$ or $3 \div 5$

4. $\frac{5}{3}$ or $5 \div 3$

5. $\frac{8}{3}$ or $8 \div 3$

6. $\frac{8}{5}$ or $8 \div 5$

Class Activity 0G: Solving Proportions by Cross-Multiplying Fractions

Read the following recipe problem:

> A recipe that serves 6 people calls for $2\frac{1}{2}$ cups of flour. How much flour will you need to serve 10 people, assuming that the ratio of people to cups of flour remains the same?

One familiar way to solve this problem is by setting up and solving a proportion as follows: First, we let x be the amount of flour we need to serve 10 people. Then we set two fractions equal to each other:

$$\frac{x}{10} = \frac{2\frac{1}{2}}{6}$$

In setting these fractions equal to each other, we may say "x is to 10 as $2\frac{1}{2}$ is to 6." Next, we "cross-multiply" to obtain the equation

$$6 \cdot x = 10 \cdot 2\frac{1}{2}$$

Finally, we solve for x by dividing both sides of the equation by 6. Therefore,

$$x = \frac{10 \cdot 2\frac{1}{2}}{6} = \frac{10 \cdot \frac{5}{2}}{6} = \frac{25}{6} = 4\frac{1}{6}$$

and we see that $4\frac{1}{6}$ cups of flour are needed to serve 10 people.

This class activity will help you understand the rationale for this method of solving proportions.

1. In the solution we just found, we worked with two fractions:

$$\frac{x}{10} \quad \text{and} \quad \frac{2\frac{1}{2}}{6}$$

   Interpret the meaning of these fractions in terms of the recipe problem at the beginning of this activity. Explain why these two fractions should be equal.

2. After setting two fractions equal to each other, the next step in solving the proportion was to “cross-multiply.” Why does it make sense to cross-multiply? What is the rationale behind the procedure of cross-multiplying?

3. In the preceding solution, we set up the proportion

$$\frac{x}{10} = \frac{2\frac{1}{2}}{6}$$

What is another way to set up a proportion so that the unknown amount of flour, x, is in the numerator of one of the fractions? Interpret the two fractions in your new proportion in terms of the recipe problem. Use your interpretations to explain why the two fractions should be equal.

4. Now solve the recipe problem in a different way by using logical thinking and by using the most elementary reasoning you can. Explain your reasoning clearly.

Class Activity 0H: Can You Always Use a Proportion?

Sometimes a problem that looks as if it could be solved by setting up a proportion actually can't be solved that way. Before you set up a proportion to solve a problem, ask the following question about quantities in the problem: if I double one of the quantities, should the other quantity also double? If the answer is "no," then you cannot solve the problem by setting up a proportion.

1. Ken used 3 loads of stone pavers to make a circular (i.e., circle-shaped) patio with a radius of 10 feet. Ken wants to make another circular patio with a radius of 15 feet, so he sets up the proportion

$$\frac{3 \text{ loads}}{10 \text{ feet}} = \frac{x \text{ loads}}{15 \text{ feet}}$$

   Is this correct? If not, why not? Is there another proportion that Ken could set up to solve the problem? (The area of a circle that has radius r units is πr^2 square units.)

2. In a cookie factory, 4 assembly lines make enough boxes of cookies to fill a truck in 10 hours. How long will it take to fill a truck if 8 assembly lines are used? Is the proportion

$$\frac{10 \text{ hours}}{4 \text{ lines}} = \frac{x \text{ hours}}{8 \text{ lines}}$$

   appropriate for this situation? Why or why not? If not, can you solve the problem another way?

3. In the cookie factory of problem 2, how long will it take to fill a truck if 6 assembly lines are used? (If you get stuck here, move on to the next problem and come back.)

4. Robyn used the following reasoning to solve the previous problem:

> Since 4 assembly lines fill a truck in 10 hours, 8 assembly lines should fill a truck in half that time, namely in 5 hours. Since 6 assembly lines is halfway between 4 and 8, it ought to take halfway between 10 hours and 5 hours, or $7\frac{1}{2}$ hours, to fill a truck.

Robyn's reasoning seems quite reasonable, but is it really correct? Let's look carefully.

Fill in the following table by using logical thinking about the assembly lines:

# of assembly lines	**# of hours to fill a truck**
1	
2	
4	10 hours
8	
16	
32	

Now apply Robyn's reasoning again, but this time to 1 assembly line versus 32. Sixteen assembly lines is approximately halfway between 1 and 32. But is the number of hours it takes to fill a truck using 16 assembly lines approximately halfway between the number of hours it takes to fill a truck using 1 assembly line versus using 32 assembly lines?

What can you conclude about Robyn's reasoning?

Class Activity 0I: (+) The Consumer Price Index

Sometimes news reports compare how much was spent on items in the past and now. For example, a report in *Newsweek* magazine on January 24, 2005, published a table like the following one, listing costs of recent presidential inaugurals.

Inaugural Price Tags (Amount spent, adjusted to 2004 $)		
Nixon	1973	$17 million
Carter	1977	11
Reagan	1981	34
Reagan	1985	35
Bush	1989	46
Clinton	1993	30
Clinton	1997	35
Bush	2001	43
Bush	2005	40 (goal)

1. Why do you think the table shows the amounts spent on inaugurals in 2004 dollars instead of the actual amounts spent?

   How do you think the table would be different if the actual dollar amounts spent on inaugurals were listed instead of 2004 dollars?

The following table shows the value of the CPI for some selected years:

Consumer Price Index (selected years)							
year	**CPI**	**year**	**CPI**	**year**	**CPI**	**year**	**CPI**
1940:	14.0	1960:	29.6	1980:	82.4	2000:	172.2
1950:	24.1	1970:	38.8	1990:	130.7	2004:	188.9

2. What salary in the year 2000 would have had the same buying power as \$20,000 did in 1950?

3. If a gallon of milk cost \$3 in 2000, what should it have cost in 1940, if you use the CPI to adjust for inflation?

   Does this necessarily give you the actual cost of a gallon of milk in 1940? Why not?

4. If a certain item cost \$2 in 1980 and \$3.50 in 2000, would it be reasonable to argue that its price had gone down from 1980 to 2000? How so?

5. In both 1980 and 2000, gasoline cost about \$1.20 per gallon. Put the price of gasoline in both years into 2000 dollars and compute the percent by which inflation adjusted gasoline prices fell from 1980 to 2000.