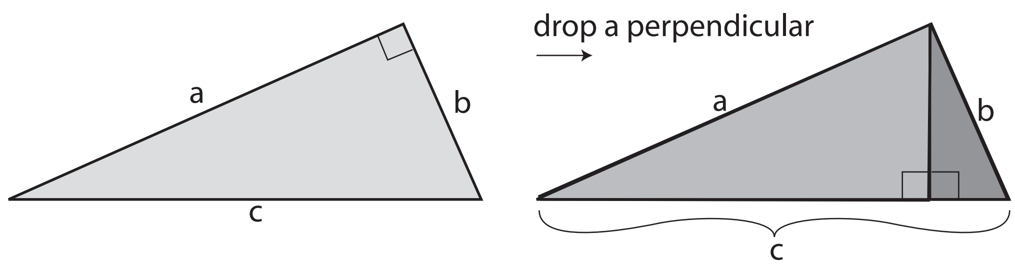
Class Activity 0A: A Scaling Proof of the Pythagorean Theorem

This activity will help you use similar shapes to prove the Pythagorean theorem.

Remember that the Pythagorean theorem says that for any right triangle with short sides of length a and b, and hypotenuse of length c,

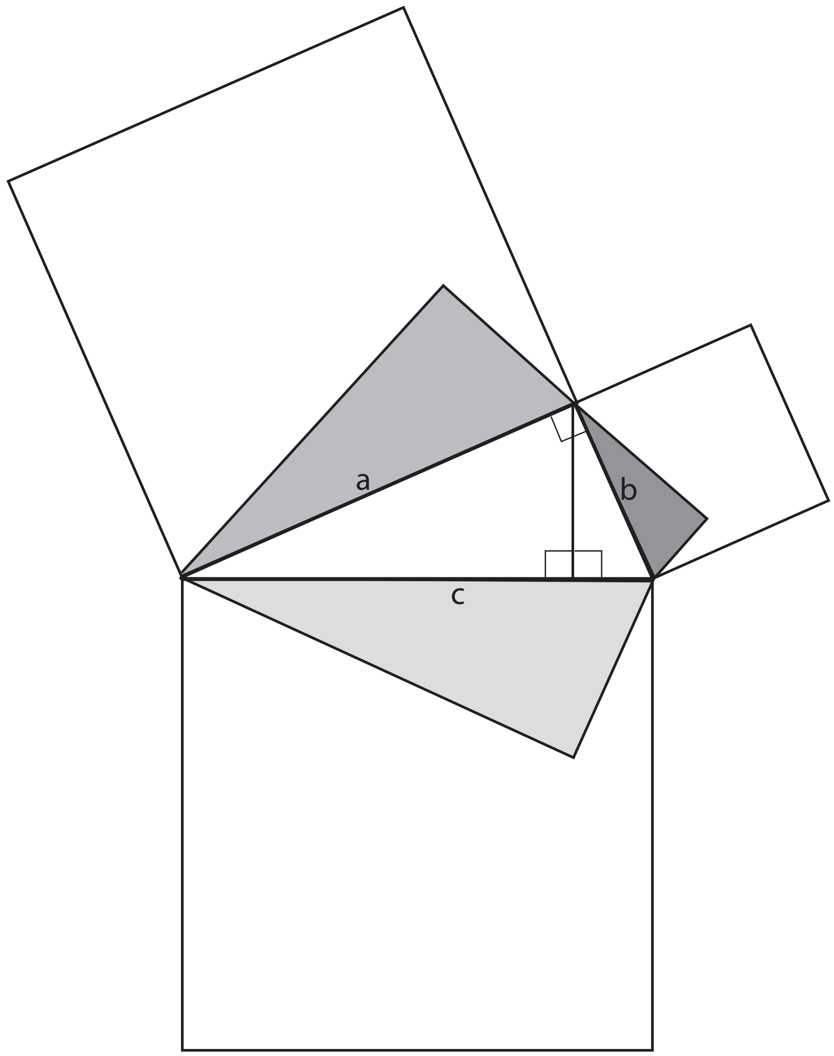
$$a^2 + b^2 = c^2$$

1. Given any right triangle, such as the next triangle on the left, drop the perpendicular to the hypotenuse, as shown on the right.



Use angles to explain why the two smaller right triangles on the right are similar to the original right triangle. (Do not use any actual measurements of angles because the proof must be general — it must work for *any* initial right triangle.)

2. Now view each of the three right triangles from part 1 (the original triangle and the two smaller ones) as taking up a percentage of the area of the square formed on its hypotenuse, as in the next figure. Why must each triangle take up the same percentage of its square?



3. Let $P\%$ be the percentage of part 2. Express the areas of the three triangles in terms of $P\%$ and then explain why

$$P\% \cdot a^2 + P\% \cdot b^2 = P\% \cdot c^2$$

4. Use part 3 to explain why

$$a^2 + b^2 = c^2$$

thus proving the Pythagorean theorem.