Why Aren't There Any Giant Snails?

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Why aren't there any snails that are as big as a dog or a person or an elephant? After all, there are plenty of large animals, such as whales and giant octopuses, and in the past there were large dinosaurs and giant sloths. Why not giant snails too? There may be many reasons why there have never been any giant snails (as far as we know), but this essay will explore some mathematical facts that have perhaps played a role in limiting the size of snails, and should also play a role in limiting the sizes of virtually all animals. The idea that there are difficulties associated with scaling up objects goes at least as far back as to the great scientist Galileo (1564-1642), who examined why larger objects require much sturdier supports than smaller ones. See
`http://galileoandeinstein.physics.virginia.edu/tns.htm`

First, let's assume that snails are more or less cube-shaped (as in Figure 1). The amount of food that a snail should have to eat ought to be roughly proportional to the volume of the snail. So a snail that has twice the volume of another, smaller snail ought to have to eat twice as much as the smaller snail. And a snail that has 3 times the volume of another, smaller snail ought to have to eat 3 times as much as the smaller snail. Snails get their food through their "foot," by creeping along and sucking up nutrients as their foot slides over the slimy surface of objects. So how much food does a snail get through its foot? Assuming a uniform environment, where all surfaces have an equal amount of nutritious (for snails) slime covering them, the amount of food that a snail gets should be proportional to the area of the snail's foot. So a snail whose foot has twice the area of a smaller snail's foot should get twice as much food as the smaller snail (if all other things are equal). But the amount of food a snail gets should also be proportional to the speed at which the snail slithers along. So a snail that slithers along at twice the speed of another slower snail should get twice as much food as the slower snail (again, assuming all other things are equal).

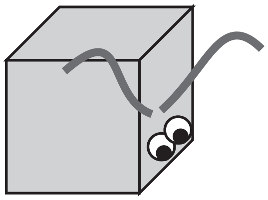

Figure 1: A Cube-Shaped Snail

Now let's imagine a collection of larger and larger snails. The first snail is a small one. The second snail is twice as long, twice as wide, and twice as tall as the first one; the third snail is three times as long, three times as wide, and three times as tall as the first one; and so on (as in Figure 2). Think about how the areas of the feet of these snails will compare to the area of the foot of the first snail and think about how the volumes of these snails will compare to the volume of the first snail. To do so, think about, and experiment with, cubes that are twice as wide, long, and high or three

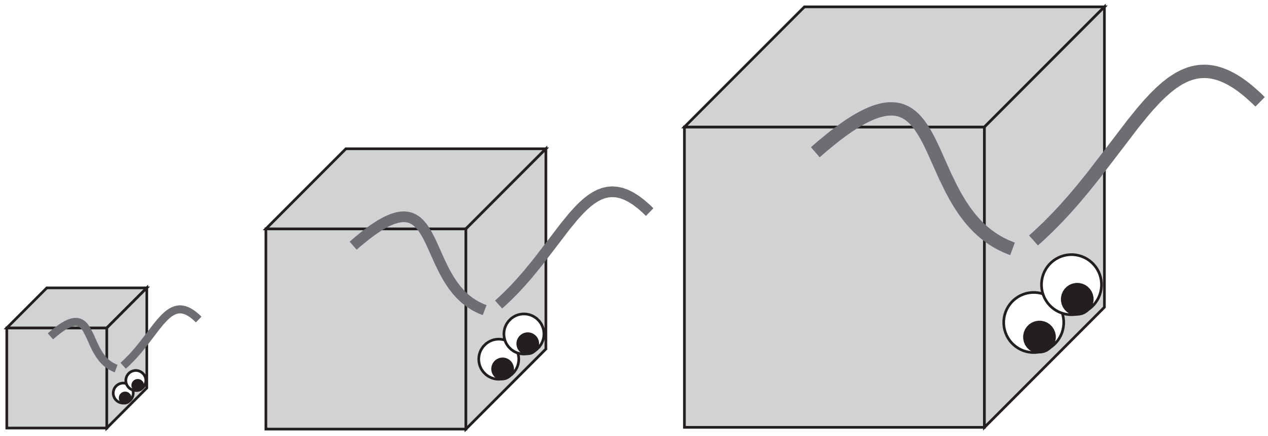

Figure 2: Larger and Larger Snails

times as wide, long, and high, or 4 times as wide, long, and high as an original cube. How do the areas of the faces of these cubes compare to the area of a face of the original cube? How do the volumes of the cubes compare to the volume of the original cube?

The table below summarizes how the foot areas and volumes of the larger snails are related to those of the first snail.

	compared to first snail		
length, width, height	2 times	3 times	4 times
foot area	4 times	9 times	16 times
volume	8 times	27 times	64 times

What are the patterns in the table? If a snail was 10 times as long, wide, and tall as the first snail, what could you say about its foot area and its volume? What if the snail was n times as long, wide, and tall as the first one? Think about these questions before you read on.

In general, a cube that is n times as long, wide, and tall as a smaller cube has a volume that is n^3 times as much as the smaller cube, and the area of a face of the larger cube is n^2 times as much as the area of a face of the smaller cube.

What do these considerations tell us? How much food should the second snail have to eat compared to the first snail? But how much food will the second snail get compared to the first snail if it moves at the same speed? Will that work? If not, how fast will the second snail have to go compared to the first snail? Answer these questions for the third and fourth snail as well. What can we conclude about how fast these snails will have to move?

A snail that is n times as wide, long, as a smaller snail will have a volume that is n^3 times the volume of the smaller snail, and so will need n^3 times as much food as the smaller snail. But the area of the foot of the larger snail is only n^2 times the area of the foot of the smaller snail. So to compensate, the larger snail would have to move n times as fast as the smaller snail in order to get the right amount of food. Eventually, if n is large enough, this will simply not be possible, and so there can't be snails beyond a certain size if all of our considerations about snails above hold.

This discussion about snails illustrates an important way that mathematics is used. To analyze a situation, we model it by making some simplifying assumptions so that we can apply some mathematics we know. In our snail example, we assumed that snails are roughly shaped like cubes and that the foot of a snail is roughly like the face of a cube. By simplifying the situation and by making some assumptions that seem reasonable, we can apply mathematics to analyze the situation and to develop a rationale for something we observe. When children explore patterns in surface areas and volumes of cubes, they are probably not thinking about applying these considerations to snails. But even the most abstract theoretical mathematics sometimes has surprising applications. And conversely, almost anything in life, even a snail, can invite us to think mathematically.