Sample test problems for Mathematics for Elementary Teachers by Sybilla Beckmann

Sample Test Problems for Chapter 9, Sections 9.4 – 9.13

1. Use *only* the formula for areas of rectangles and the moving and combining principles about area to determine the area of the shaded shape in Figure 1. Explain your method.

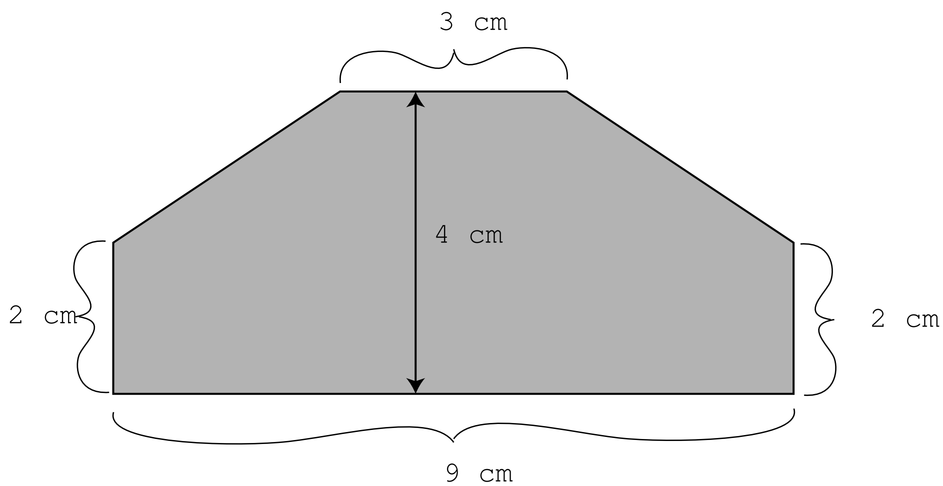

Figure 1: A Shape

2. Determine the area of the quadrilateral in Figure 2 using only the principles we have studied about area and the area formula for rectangles (do not use any other area formulas). Explain your reasoning. Adjacent dots on the grid are 1 cm apart.

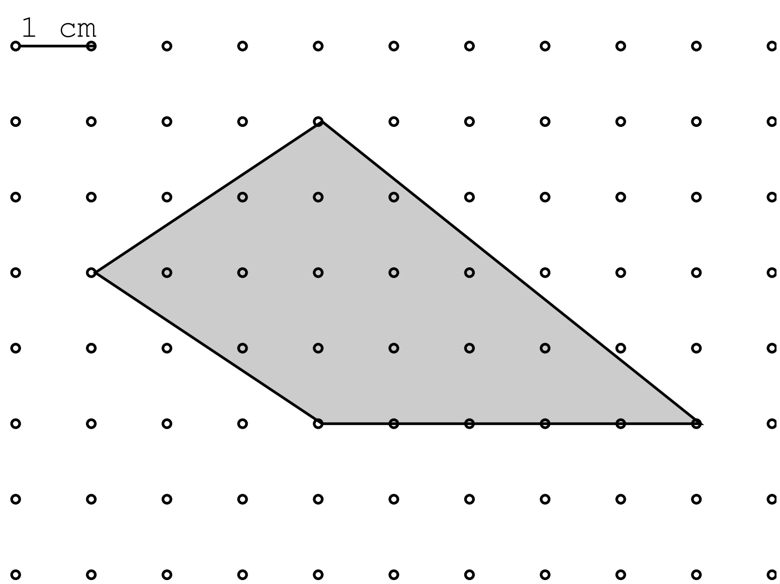

Figure 2: A Quadrilateral

3. Determine the area of the shaded quadrilateral in Figure 3 in *two different ways.* Explain your reasoning both times.

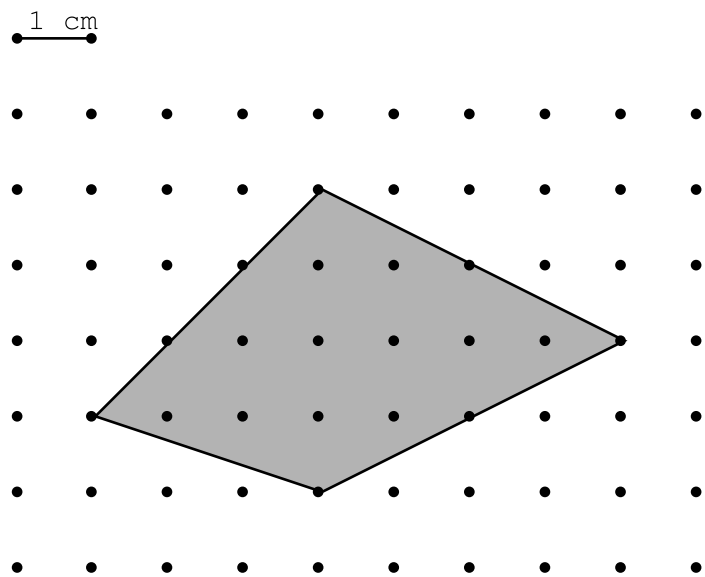

Figure 3: A Quadrilateral

4. Determine the area of the shaded triangle in Figure 4. Indicate your reasoning.

5. Use Figure 5 to explain why $a^2 + b^2 = c^2$, where a and b are the lengths of the short sides of a right triangle, and c is the length of the triangle's hypotenuse. (You may assume that all shapes that look like squares really are squares, and that in each picture, all four triangles with side lengths a, b, c are identical right triangles.)

6. What is the longest pole that can fit in a box that is 4 feet wide, 3 feet deep, and 5 feet tall? Explain.

7. Determine the area of the parallelogram in Figure 6 using only the principles we have studied about area and the area formula for rectangles (do not use any other area formulas). Explain your reasoning. Adjacent dots on the grid are 1 cm apart.

8. Determine the area of the triangle in Figure 7 using only the principles we have studied about area and the area formula for rectangles (do not use any other area formulas). Explain your reasoning. Adjacent dots on the grid are 1 cm apart.

9. Determine the area of the shaded shape in Figure 8. Explain your reasoning briefly.

10. Give a clear and thorough explanation for why the area of the triangle in Figure 9 is $\frac{1}{2}b \times h$ square units for the given choices of base b and height h.

11. Give a clear and thorough explanation for why the area of the triangle in Figure 10 is $\frac{1}{2}b \times h$ square units for the given choices of base b and height h.

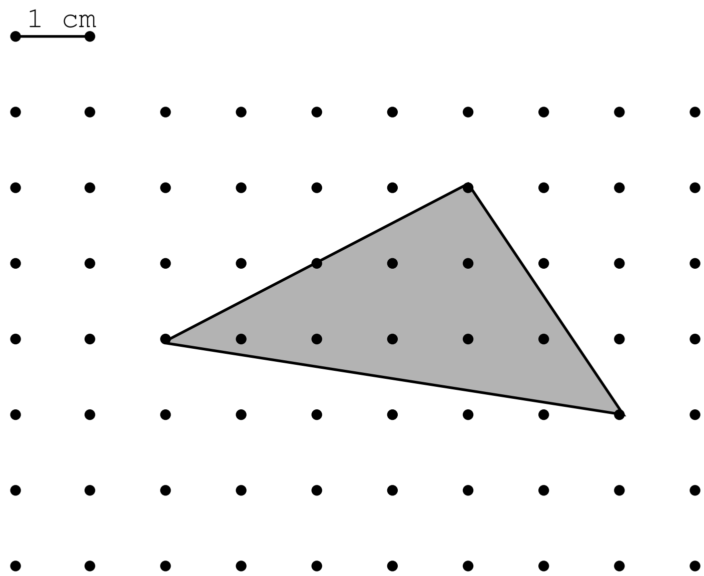

Figure 4: A Triangle

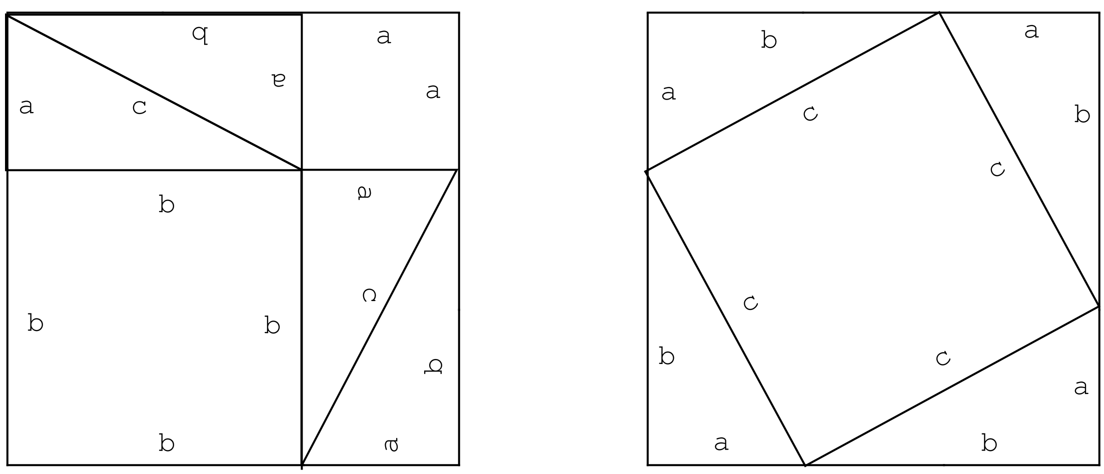

Figure 5: Shapes Forming Identical Large Squares

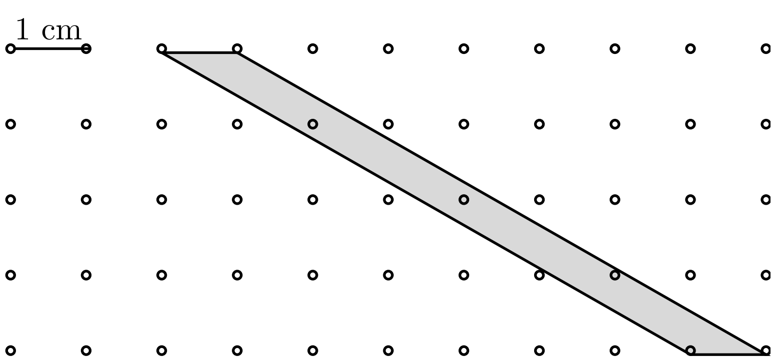

Figure 6: A Parallelogram

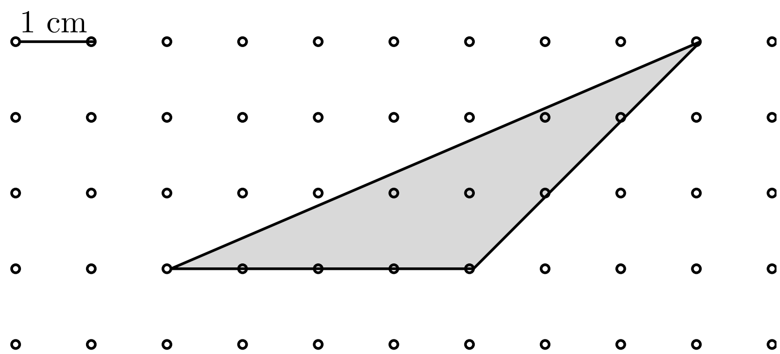

Figure 7: A Triangle

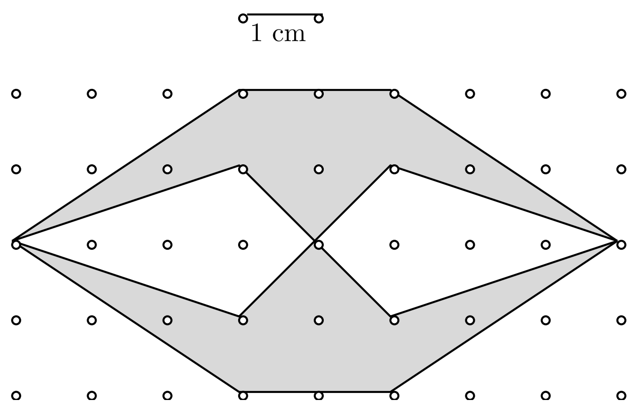

Figure 8: Determine the Area

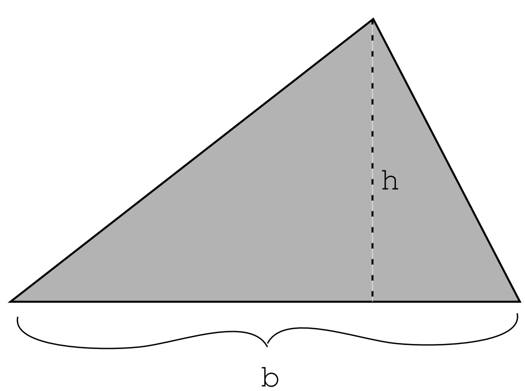

Figure 9: A Triangle

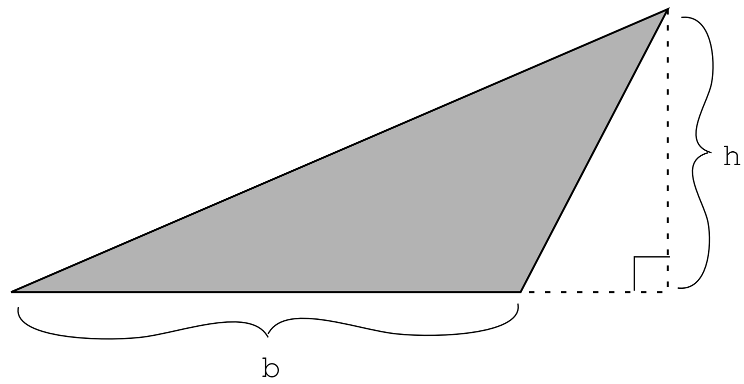

Figure 10: A Triangle

12. Explain clearly why there can be no formula for areas of parallelograms that is *only* in terms of the lengths of the sides of the parallelogram.

13. In the gym there is a round metal pole that is 8 inches in diameter. The gym teacher wants to wrap the pole tightly with 1 inch thick rope from the ground up to a height of 6 feet. (The rope will be wound around the pole over and over until it reaches a height of 6 feet.) About how much rope will the gym teacher need? Explain your reasoning.

14. Given that the circumference of a circle of radius r units is $2\pi r$ units, explain how to subdivide and rearrange a circle of radius r units in order to show why the area of this circle is πr^2 square units.

15. Use the moving and combining principles about area to determine the surface area of a 2 inch tall tin can that has diameter 3 inches. (The tin can has a top and bottom.) Explain your reasoning.

16. Suppose you take a rectangular piece of paper, roll it up, and tape two ends together, without overlapping them, to make a tube. If the tube is $8\frac{1}{2}$ inches long and has a diameter of $3\frac{1}{2}$ inches, then what were the length and width of the piece of paper? Explain your reasoning.

17. A garden path surrounds a circular garden, as shown in Figure 11. The garden path is 5 feet wide all the way around and the distance around the outside of the garden path is 80 feet.

    (a) What is the area of the garden path? Explain.

    (b) What is the area of the garden (inside the path)? Explain.

18. A concrete patio will be made in the shape of a 12 foot by 12 foot square with half-circles attached at two opposite ends, as pictured in Figure 15. What is the area of this patio?

19. The distance around a piece of property is 5.3 miles. With only this information about the property, can you determine the area of the property? If so, explain how, if not, explain why not.

20. The distance around a piece of property is 5.3 miles.

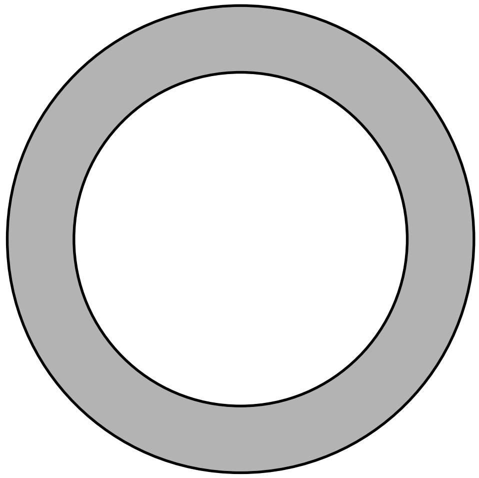

Figure 11: A Garden Path

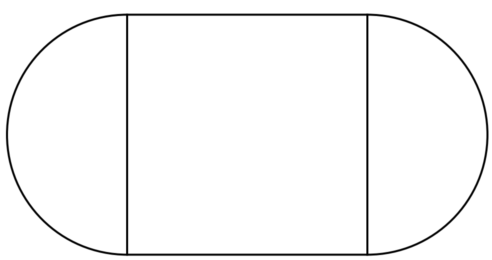

Figure 12: A Patio

(a) If you don't know anything about the shape of the property, then what is the largest area that the property could have? Explain.

(b) If you know that the property is in the shape of a rectangle, then what is the largest area that the property could have? Explain.

(c) If you know that the property is in the shape of a rectangle, then what is the range of possible areas that the property could have? Explain.

(d) Is it possible that the area of the property is 1 square mile? Explain.

21. Sam wants to find the area of an irregular shape. Sam cuts a piece of string to the length of the perimeter of the shape. Sam measures that the string is about 40 cm long. Sam then forms his string into a square on top of centimeter graph paper. Using the graph paper, Sam determines that the area of his string square is about 100 cm^2. Sam says that therefore the area of the irregular shape is also 100 cm^2. Is Sam's method for determining the area of the irregular shape valid or not? Explain. If the method is not valid, what can you determine about the area of the irregular shape from the information that Sam has? Explain.

22. (a) On the $\frac{1}{4}$ inch graph paper in Figure 13 (meaning that adjacent grid lines are $\frac{1}{4}$ inch apart), draw two different rectangles that have perimeter $5\frac{1}{2}$ inches.

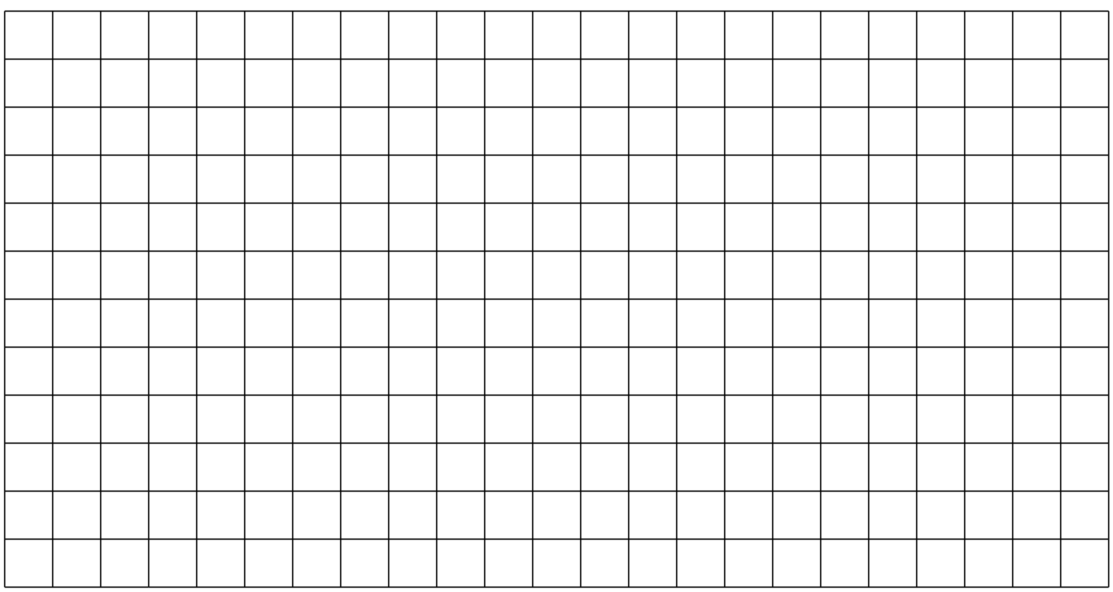

Figure 13: Quarter-Inch Graph Paper

(b) Without using a calculator, determine the areas of your rectangles. Show your calculations, or explain briefly how you determine the areas.

23. (a) Describe a concrete way to demonstrate that many different shapes can have the same area.

(b) Describe a concrete way to demonstrate that many different shapes can have the same perimeter.

(c) Describe a concrete way to demonstrate that many different solid shapes can have the same volume.

24. Suppose you have a paper cup floating in a measuring cup that contains water. When the paper cup is empty, the water level in the measuring cup is at 250 ml. When you put 9 quarters in the paper cup (which is still floating), the water level in the measuring cup is at 300 ml. What information about a quarter can you deduce from this experiment? Explain.

25. The front (and back) of a greenhouse have the shape and dimensions shown in Figure 14. The greenhouse is 40 feet long and the angle at the top of the roof is 90°. Determine the volume of the greenhouse in cubic feet. Explain your solution.

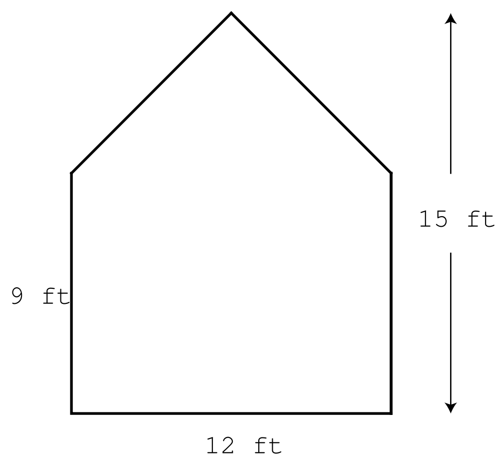

Figure 14: A Greenhouse

26. A concrete patio will be made in the shape of a 12 foot by 12 foot square with half-circles attached at two opposite ends, as pictured in Figure 15. If the concrete will be 4 inches thick, then how many cubic yards of concrete will be needed for the patio? Explain your solution.

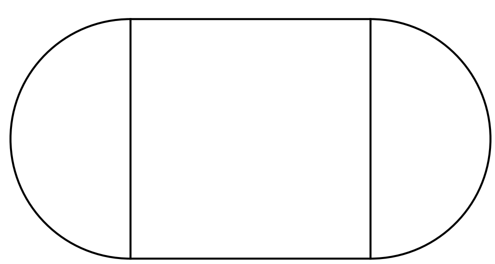

Figure 15: A Patio

27. Caulking is often used to seal around bathtubs and showers in order to make them waterproof. When you squeeze caulking out of a tube, it comes out in the shape of a (very long) cylinder whose diameter is the diameter of the hole in the tube where the caulking comes out. Suppose that a tube of caulking has a hole of diameter $\frac{1}{8}$ of an inch and suppose that you can use the tube of caulking to seal 50 feet worth of edges around bathtubs and showers. How many cubic inches of caulking must have been in the tube when it was full? Explain your answer.

28. The front and back of a storage shed are shaped like half-circles of diameter 16 feet, as shown in Figure 16. The shed is 25 feet long.

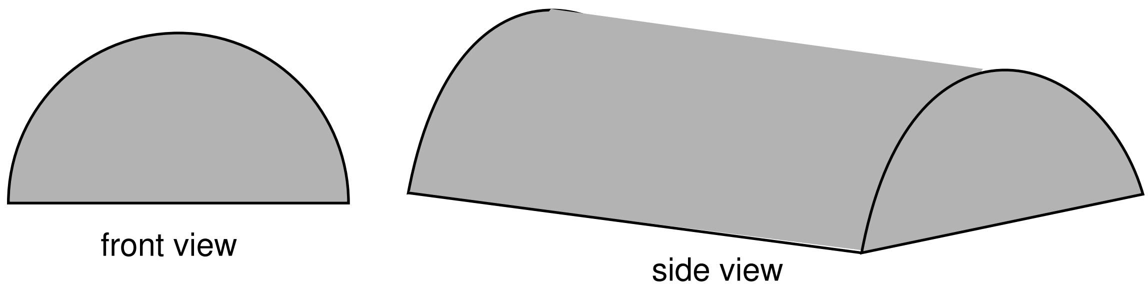

Figure 16: A Storage Shed

(a) Determine the volume of the storage shed, indicating your reasoning.

(b) The top of the storage shed is to be covered with plastic sheeting (not including the front and back). What size piece of plastic is needed to cover the shed? Explain your reasoning.

29. Why is there a $\frac{1}{3}$ in the volume formula for a pyramid? Give some idea for where the $\frac{1}{3}$ comes from.

30. A sand and gravel company has a cone-shaped pile of sand. The company measures that the distance around the pile of sand at the base is 85 feet and the "slanted" distance from the edge of the pile at ground level to the top of the pile is 25 feet. Determine the volume of sand in the cone-shaped pile, explaining your solution.

31. Kelsey made a scale model of one of the Egyptian pyramids. Kelsey's model is 10 cm tall. The actual Egyptian pyramid is 1000 times as tall as Kelsey's model.

(a) Originally, the Egyptian pyramid had been covered with a thin layer of alabaster, which has since washed away. Kelsey covered her pyramid model with shiny paper in order to simulate the alabaster. Kelsey determined that she used approximately 200 square centimeters of shiny paper to cover her pyramid. Approximately how many square meters of alabaster had been needed to cover the actual Egyptian pyramid? Explain.

(b) Kelsey filled her pyramid with rice and determined that her pyramid holds about 300 ml. What is the volume of the original Egyptian pyramid in cubic meters? Explain.

32. A large iguana can be 7 feet long and weigh 16 pounds. Suppose that from excavated bones, a dinosaur was found to have been 25 feet long and proportioned like a large iguana. Give a good estimate for the weight of the dinosaur. Explain your reasoning.

33. A typical adult male gorilla is about $5\frac{1}{2}$ feet tall and weighs about 400 pounds. King Kong was supposed to have been about 20 feet tall. Assuming that King Kong was proportioned like a typical adult male gorilla, approximately how much should King Kong have weighed? Explain your reasoning.