STUDY GUIDE

For Ph.D. Written Qualifying Examinations

Department of Mathematics

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This document provides study guides for subjects in which written qualifying exams are given. Each guide lists topics that a student should know for the corresponding examination. An attempt has been made to put these topics in coherent order; occasionally, a theorem might be placed under several headings. The study guides are not intended as syllabi for corresponding graduate courses, and a graduate course might omit some of the topics in the study guide and include others. The topics in these study guides and related material should be mastered before attempting the qualifying exams.

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Distribution Requirements

The Analysis exam lasts 4 hours with a break in the middle. All other exams are three hours in length. The corresponding semester courses in each area are designed to prepare the student for qualifying exams. Written qualifying exams are offered every year in August, during the week before the start of classes, and in January during the week before the start of classes. Syllabi, and old copies of qualifying exams, are available from Jennifer Peterson in room 434C for students to use in studying for qualifying exams.

Written qualifying exams offered are Analysis, Algebra, Topology, Probability, and Numerical Analysis. All Ph.D. candidates must pass three written qualifying exams including Analysis and either Algebra or Topology.

Study Guide for Algebra Exams

Foundations:

Zorn's Lemma and its uses in various existence theorems such as that of a basis for a vector space or the algebraic closure of a field.

References: [1,4]

Group Theory:

- subgroups
- quotient groups
- Lagrange's Theorem
- fundamental homomorphism theorems
- automorphisms and group actions with applications to the structure of groups such as the Sylow Theorems
- group constructions
  - such as:
  - free groups
  - generators and relations
  - direct and semi-direct products
- structures of special types of groups
  - such as:
  - p-groups
  - solvable groups
  - dihedral, symmetric and alternating groups
  - linear groups (GL_n , SL_n, O_n, U_n, etc.)
- The simplicity of A_n, for $n \geq 5$

References: [1,3,5].

Theory of Rings and Modules

- basic properties of ideals and quotient rings
- fundamental homomorphism theorems for rings and modules
- characterizations and properties of special domains
  - such as:
  - PIDs, UFDs and Euclidean domains
  - the classification of finitely generated modules over a PID
  - applications to the structure of finitely generated abelian groups and canonical forms of matrices
  - Noetherian rings and modules

References: [1,3,4,5].

Field Theory:

algebraic and transcendental extensions of fields
fundamental theorems of Galois theory and of elementary symmetric functions
polynomials of small degree
properties of finite fields
separable and inseparable extensions
computations of Galois groups of cyclotomic polynomials
solvability of polynomials by radicals

References; [1,3,5]

Linear Algebra:

eigenvalues and eigenvectors
Cayley-Hamilton Theorem
canonical forms for matrices
determinants
dual spaces
tensor products of vector spaces
finite-dimesional spectral theorem

References: [1,2,5]

References

[1] Thomas W. Hungerford, *Algebra*, Springer, New York, 1974.
[2] Kenneth Hoffman and Ray Kunze, *Linear Algebra*, Prentice-Hall, 1961.
[3] Nathan Jacobson, *Basic Algebra 1*, W.H. Freeman, San Francisco, 1974.
[4] Nathan Jacobson, *Basic Algebra 2*, W. H. Freeman, San Francisco, 1980.
[5] Serge Lang, *Algebra*, Addison Wesley, Reading Mass., 1970

Study Guide for Analysis Exams

I. Topics in Calculus

Continuity and differentiation in one and several variables
Taylor's Theorem
Riemann-Stieltjes integrals
Inverse and implicit function theorems

References: [2.6]

II. Structure of the Real Number Systems and Metric Spaces

Compact metric spaces
Heine-Borel and Bolzano-Weierstrass Theorems
Baire category theorem
Convergence of sequences and series
Uniform convergence
Uniformly continuous functions
Equicontinuity and Arzela-Ascoli theorem

References: [2,6]

III. Topics in Measure and Integration

Measures on σ-algebras
Borel and Lebesgue measures
Measurable functions
Integration theory
Convergence theorems (Fatou lemma, monotone and dominated convergence theorems)
Properties of L^p spaces (including ℓ^p)
Riesz representation theorems for $C_c(X)$ and L^p
Lebesgue decomposition theorem
Radon-Nikodym theorem
Product measures
Fubini and Tonelli theorems

References: [5,6,7]

IV. Cauchy's Theorem and its consequences

Cauchy's theorem and integral formula
Morera's theorem
Winding numbers, the argument principle
Rouche's theorem

The open mapping theorem
Maximum modulus principle
Liouville's theorem
Fundamental theorem of algebra

References: [1,4]

V. Elementary Properties of Analytic Functions

The Cauchy-Riemann equations
Examples of analytic functions
Normal families
Uniform convergence of analytic functions
Radius of convergence
Explicit calculation of Taylor and Laurent expansions

References: [1,4]

VI. Singularities

Removable singularities, poles, essential singularities
Residue theorem and contour integration
Casorati-Weierstrass theorem

References; [3,4]

VII. Conformal Mapping

Linear fractional transformations
Schwarz's lemma
Reflection principle

References: [1,4 (vol.II].

References

[1] L. Ahlfors, *Complex Analysis*, McGraw-Hill
[2] R. Buck, *Advanced Calculus*, 3rd edition
[3] J. Conway, *Functions of One Complex Variable*, 2nd edition, Springer
[4] E. Hille, *Analytic Function Theory*, Vols. 1 and 2, Ginn
[5] H. Royden, *Real Analysis*, 3rd edition
[6] W. Rudin, *Principle of Mathematical Analysis*, 3rd edition
[7] W. Rudin, *Real and Complex Analysis*, 3rd edition

Study Guide for Numerical Analysis Exams

Number systems and errors in digital computation, machine unit round off error. References: [1,2]

Numerical solution of nonlinear equations. References: [1,2]

Interpolation theory and applications. References: [1,2]

Numerical integration in one or more dimensions. References: [1,2]

Spline theory and applications in computer graphics. References: [1,2,3,4]

Numerical differentiation. References: [2,5]

Remainder theory and Peano's Theorem. References: [2,5]

Approximation theory and applications. References: [1,2]

Direct and iterative methods for linear systems. References: [1,2]

Algebraic eigenvalue problem. References:[1,2]

Numerical solution of systems of ordinary differential equations. References: [1,2]

Numerical methods for boundary value problems involving ordinary differential equations. References: [1]

Solution of systems of nonlinear equations. References: [1,2]

Optimization and nonlinear least squares techniques. References: [1,2]

References

[1] Burden, R.L. and Faires, J.D., *Numerical Analysis*, 4th edition, PWS Publishers, 1985
[2] Atkinson, K.E., *An Introduction to Numerical Analysis*, 2nd edition, John Wiley and Sons, 1989
[3] Brodies, K.W. (Ed), *Mathematical Methods in Computer Graphis and Design*, Academic Press, 1980
[4] Swan, T., *Mastering Turbo Pascal 5.5*, Hayden Books, 1989
[5] Davis, P., *Interpolation and Approximations*, Blaisdell, 1965

Study Guide for Probability Theory Exams

MATHEMATICAL FOUNDATION OF PROBABILITY IS ASSUMED: Random variable (r.v.s.), expectation and higher moments of r.v.s., Fatou's lemma, monotone and dominated convergence theorems; inequalities of Markov, Chebyshev, Holder, Minkowskii, and Jenson.

Convergence; Distribution Functions and Characteristic Functions:

Weak convergence of probability measures, Alexandrov theorem,
tightness and weak compactness, Prohorov theorem
Infinitely divisible distribution and Levy-Khintchine representation.

References: [1,3,4,5]

Laws of Large Numbers

Sums of independent r.v.s., Khinntchine-Kolmogorov theorem
Kolmogorov's Three-series and Two-series theorems
Weak and Strong laws of large numbers

References: [1,2,3,4,5]

Central Limit Theorems

Various central limit theorems and rates of convergence
Convergence in distribution to infinitely divisible distributions

References: [1,2,4,5]

Discrete-time Martingales

Martingales and semimartingales
Doob's inequalities (including upcrossing inequality)
Optional sampling and convergence theorems

References: [1,2,4,5]

References

[1] K.L. Chung: *A Course in Probability Theory*, 2nd Edition, Academic Press, N.Y., 1978.
[2] Y.S. Chow and H. Teicher: *Probability Theory*, 2nd Edition, Springer Verlag, N.Y., 1988.

[3] B.V. Gnedenko and A.N. Kolmogorov: *Limit Distributions for Sums of Independent Random Variables*, 2nd Edition, Addison-Wesley, Massachusetts, 1961.
[4] R.G. Laha and V.K. Rohatgi: *Probability Theory*, John Wiley, N.Y., 1979.
[5] A.N. Shiryayev: *Probability*, Springer-Verlag, N.Y., 1984.

Study Guide for Topology Exams

General Topology

topological spaces and continuous functions
product and quotient topology
connectedness and compactness
Urysohn lemma
complete metric spaces and function spaces

References: [2]

Algebraic Topology

fundamental group
vanKampen's theorem
classifications of surfaces
classifications of covering spaces
homology:
- simplicial, singular and cellular: computations and applications
- degree of maps
- Euler characteristics
- Lefschetz fixed point theorem

References: [1,3]

The weight of topics on the exam should be about 1/3 general topology and 2/3 algebraic topology.

References

[1] W. Massey, *Algebraic Topology: An Introduction*, Springer Verlag, 1977.
[2] J. Munkres, *Topology, A First Course*, Prentice-Hall, 1975.
[3] J. Munkres, *Elements of Algebraic Topology*, Addison-Wesley, 1984.