University of Georgia   Math 1113
Department of Mathematics   **Test 2**   Spring 2023

By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else during the exam:

Name (sign): _____   Name (print): _____

Student Number: _____

Instructor's Name: _____   Class Time: _____

- If you need extra space use the last page.

- Please show your work. **An unjustified answer may receive little or no credit.**

- If you make use of a theorem to justify a conclusion then state the theorem used by name.

- Your work must be **neat**. If I can't read it (or can't find it), I can't grade it.

- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.

- Please turn off your mobile phone.

- A calculator is not necessary, but numerical answers should be given in a form that can be directly entered into a calculator.

- Common identities:

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta),$$
$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta).$$

Problem Number	Points Possible	Points Made
1	0	
2	21	
3	18	
4	18	
5	10	
6	18	
7	15	
Total:	100	

1. [2 Bonus] Common Knowledge: Who is the better sprinter Elisa Balsamo, Charlotte Kool, or Lorena Wiebes?

2. Determine all of the values of x for each question below that satisfy the given equation. If no values of x satisfy the equation provide a brief justification as to how you arrived at your conclusion.

_____ (a) [5 pts] $\ln(x - 1) = 3$.

(b) [5 pts] $e^{4x+1} = 8$.
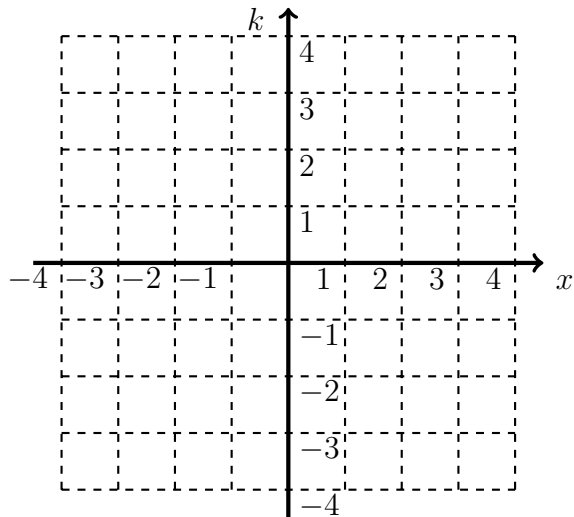
_____

(c) [5 pts] $\ln(x) + \ln(x + 1) = 2$.

_____

(d) [6 pts] $3 \cdot 8^{2x+1} = 14 \cdot 6^{1-x}$.

_____

Points earned: _____
out of a possible 11 points

3. The questions below refer to the function,

$$Ra(x) = \ln(x+1) + 2.$$

(a) [6 pts] Use the axes below to draw a sketch of the graph of $Ra(x)$. **Also, state the domain and range of the function.**

(b) [6 pts] Show that the function $Ra(x)$ is one-to-one.

(c) [6 pts] Determine the inverse of the function $Ra(x)$.

4. The function $Kr(x)$ is defined by

$$Kr(x) \;=\; 3 + C \cdot e^{rx}.$$

It is known that $Kr(0) = 2.1$ and $Kr(10) = 2.9$.

(a) [6 pts] Determine the values of the constants C and r.

(b) [6 pts] What is the value of $Kr(100)$? What value does the function get close to as x gets very large?

(c) [6 pts] What is the value of x when $Kr(x) = 2.99$?

5. For each of the following expressions, simplify as indicated.

(a) [5 pts] Write the following as a single exponential:

$$\left(\frac{e^{2x+2}}{e^{4x-1}}e^{x^2}\right)^3$$

(b) [5 pts] Write the following as a single logarithm:

$$\frac{1}{2}\ln(3x+1) - 7\ln(1-x) + 1.$$

Points earned: _____
out of a possible 10 points

6. Given the approximations below determine approximations for each of the expressions that follow,

$$\log_b(3) \approx 0.458, \qquad \log_b(4) \approx 0.578, \qquad \log_b(20) \approx 1.249,$$

(a) [6 pts] $\log_b(12)$.

(b) [6 pts] $\log_b(5)$.

(c) [6 pts] $\log_b(2)$.

7. A bank offers different accounts with interest rates that are compounded monthly.

    (a) [5 pts] One account offers an interest rate of 1.3% compounded monthly. If the initial principal is $10,000 what will the balance be after four years?

    (b) [5 pts] Another account offers an interest rate of 1.1% compounded monthly. How long will it take to double the initial balance?

    (c) [5 pts] A customer deposited $20,000 into an account whose interest was compounded monthly and forgot about it. After thirty years the balance on the account was $34,000. What was the interest rate on the account?

Extra space for work. **Do not detach this page.** If you want us to consider the work on this page you should print your name, instructor and class meeting time below.

Name (print): _____ Instructor (print): _____ Time: _____