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MATH 1113 Exam 1 Review

Fall 2017

Topics Covered

Section 1.1: Rectangular Coordinate System

Section 1.2: Circles

Section 1.3: Functions and Relations

Section 1.4: Linear Equations in Two Variables and Linear Functions

Section 1.5: Applications of Linear Equations and Modeling

Section 1.6: Transformations and Graphs

Section 1.7: Analyzing Graphs of Functions and Piecewise Functions

Section 1.8: Algebra of Functions and Composition of Functions

Section 2.1: Quadratic Functions and Applications

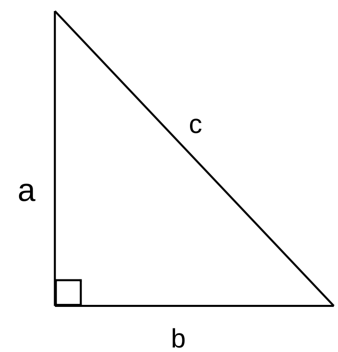
Section 1.1: Rectangular Coordinate System

The xy-Plane
The xy-plane consists of a horizontal axis (the x-axis) and a vertical axis (the y-axis).
The origin is located at the intersection of the two axes.
Locations on the xy-plane are denoted by ordered pairs (x, y)

The Pythagorean Theorem
Given a right triangle, the lengths of the sides are related by the following equation:

$$a^2 + b^2 = c^2$$

Where a and b are the sides (legs) that form the right angle and c is the hypotenuse of the triangle.



The Distance Formula
The distance d between two points (x_1, y_1) and (x_2, y_2) is derived from the Pythagorean Theorem and found by using the following equation:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

The Midpoint Formula
The midpoint of a line segment joining points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Examples

1. Given $P(-2,3)$ and $Q(8, -5)$, find
   (a) The distance $d(P.Q)$.

   (b) The midpoint of segment PQ.

(c) A point R such that Q is the midpoint of PR.

2. Find all points on the x-axis that are a distance 5 units away from $P(3,4)$.

Section 1.2: Circles

The Equation of a Circle in Standard Form

The equation of a circle with center (h, k) and radius r is:

$$(x-h)^2 + (y-k)^2 = r^2$$

Examples

3. Find an equation of the circle that has center at $C(4, -3)$ and point $A(10,5)$ on the circle. Sketch the circle in the xy-plane.

4. Find an equation of a circle that has center at $C(-2, -8)$ and is tangent to the line $x = 2$.

5. Find the center and radius of a circle with the given equation:

$$3x^2 + 24y - 18x + 3y^2 - 33 = 0$$

Section 1.3: Functions and Relations

Functions

A function from A to B is a rule of correspondence that assigns each element of A to exactly ONE element of B.

Vertical line test

A graph in the xy-plane represents a function $y = f(x)$ provided that any vertical line intersects the graph in at most one point.

Domain (the inputs)

The domain of a function $f(x)$ represents the set of all x values allowed to go into $f(x)$.

Finding the domain ***graphically***: Read the function from left to right

Finding the range ***algebraically***: Check for one of the following three 'road blocks'.

1. Exclude any values of x that would make a denominator equal to zero
2. Exclude any values of x that would make expressions under *even* roots negative.
3. Exclude any values of x that would make expressions inside log functions non-positive.

Range (the outputs)

The range of function $f(x)$ represents the set of all y values allowed to come out of $f(x)$.

Finding the range graphically: Read the function from bottom to top

Examples

6. Given the following function $f(x)$ answer the following:

$$f(x) = \frac{1}{\sqrt{x} - 7}$$

(a) Find the domain

(b) Evaluate $f(1)$, $f(-1)$ and $f(0)$

(c) Evaluate $-f(x)$, $f(x^2)$ and $[f(x)]^2$

7. What is the domain of $f(x) = (x+7)^{1/2}$

Section 1.4: Linear Equations in Two Variables and Linear Functions

The Slope-Intercept Formula
The equation of a line with slope m and y-intercept b is:

$$y = mx + b$$

The Point-Slope Formula
The equation of a line with slope m passing through the point (x_1, y_1) is:

$$y - y_1 = m(x - x_1)$$

Equations of Horizontal and Vertical Lines
The equation of a horizontal line through the point (a, b) is $y = b$. (All run, no rise)
The equation of a vertical line through the point (a, b) is $x = a$. (All rise, no run)

Parallel and Perpendicular lines
Two non-vertical lines with slopes $m_1$and m_2 are parallel if and only if

$$m_1 = m_2$$

Two non-vertical lines with slopes m_1 and m_2 are perpendicular if and only if

$$m_1 = -\frac{1}{m_2}$$

Examples

8. Find the slope of the line that passes through the point (4,7) and is perpendicular to the line$x + 6y + 4 = 0$.

9. Which of the following lines are parallel to the line $y = \frac{5}{2}x - 7$?

$$y = \frac{5}{2}x + 20 \quad 5x - 2y + 20 = 0 \quad y = \frac{2}{5}x + 20$$
$$y = -\frac{5}{2}x + 2 \quad 5x + 2y + 20 = 0$$

Section 1.5: Applications of Linear Equations and Modeling

10. Find the point P on the graph of $f(x) = \frac{1}{4x}$ such that the slope of the line through (1 , ¼) and P is – $4/3$.

11. The value of a newly purchased boat is a linear function of time. The boat was purchased for 15,000 dollars. If the value of the boat decreases to 75 percent of its purchase price in 3 years, determine the value of the boat after 7 years.

12. Kim really loves donuts. She is also concerned about her health so she wants to track how many calories a week she consumes due to her donut habit. If she lets x be the number of donuts she eats in a week, y be the number of calories consumed per week from donuts and assuming a linear relationship between x and y, answer the following.

(a) Should the slope of the line be positive, negative or zero? Why do you think so?

(b) Should the y-intercept be positive, negative or zero? Why do you think so?

13. Josh sells burritos from his food truck on weekends. If he prices them at $4.50 each, he typically can sell 200 burritos. However, for every $0.25 reduction in price, he sells an additional 20 burritos.

(a) Build a linear model for Josh's revenue as a function of price.

(b) How many burritos will Josh sell if he prices them at $6.00?

(c) Josh's goal is to sell 420 burritos in a single day. Is this feasible and at what price?

Section 1.6: Transformations and Graphs

Translations and Reflections of Functions

$y = f(x) + c$	Translate c units vertically upward
$y = f(x) - c$	Translate c units vertically downward
$y = f(x + c)$	Translate c units to the left
$y = f(x - c)$	Translate c units to the right
$y = -f(x)$	Reflect about the x-axis
$y = f(-x)$	Reflect about the y-axis
$y = cf(x)$	Stretches vertically by a factor of c
$y = f(cx)$	Stretches horizontally by a factor of c

NOTE: When making multiple movements, do all reflections first and then all translations.

Examples

14. The choices below represent instructions for translating and/or reflecting the graph of $y = f(x)$. Select the procedure for obtaining the graph of $y = -f(x) + 7$.
    (a) Reflect across the x-axis and shift 7 units downwards
    (b) Reflect across the y-axis and shift 7 units downwards
    (c) Reflect across the x-axis and shift 7 units upwards
    (d) Shift 7 units upwards and then reflect across the x-axis
    (e) Reflect across the y-axis and shift 7 units upwards

15. Point $P(1,2)$ lies on the graph of $f(x)$. The domain of $f(x)$ is $[0,10]$ and the range is $[-3,8]$.
    (a) Find the location of P on $y = -2f(4x) - 6$.

    (b) Find the domain and range for $y = -2f(4x) - 6$.

Section 1.7: Analyzing Graphs of Functions and Piecewise Functions

Piecewise Functions

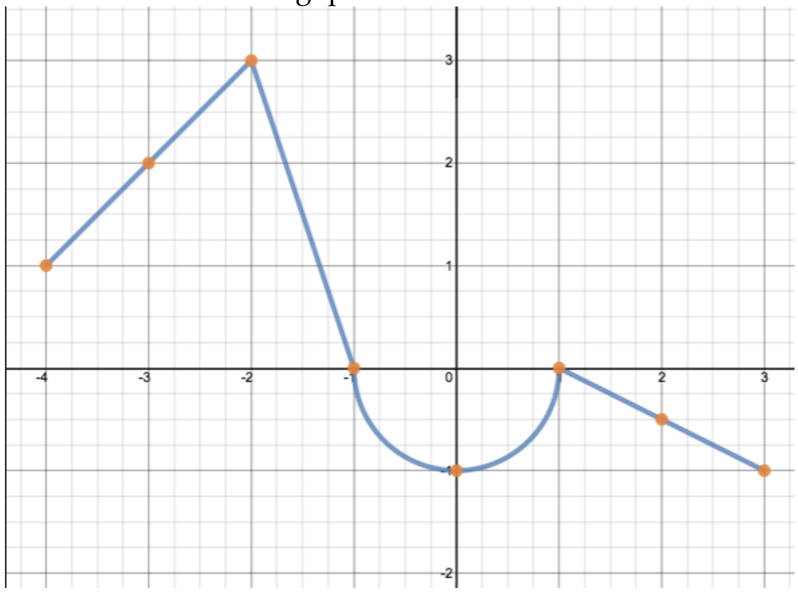
Piecewise functions are functions that are made up of pieces of other functions.

Plugging in values of x:
1. Look at the column on right side and find the interval that contains the desired value of x.
2. Use the corresponding piece of the function in the left column to evaluate.

Sketching a piecewise function
1. Draw the function from the left column using a dotted line
2. Use the corresponding interval in the right column to fill in the allowed values with a solid line
3. Remove the remaining unused part of the function
4. Repeat for each piece of the function.

Examples

16. Use the graph below to answer the following questions.



(a) What is the domain of the function?

(b) What is the range of the function?

(c) Find an equation for $f(x)$?

17. Christy, Tim and Dory are taking a road trip to see their favorite band Family and Friends. They leave Athens at noon and drive for 4 hours at 60mph, they stop for lunch for an hour, get back on the road and drive 50mph for 2 more hours and when they hit town there is a traffic jam so only go 20mph for the remaining half hour.

(a) Sketch a function of distance vs. time for their journey.

(b) What is the formula for the distance function?

(c) What time do they arrive?

(d) How far was their total drive?

(e) How far had they traveled by 5:30PM?

Section 1.8: Algebra of Functions and Composition of Functions

Algebraic Properties of Functions

Let $f(x)$ and $g(x)$ be functions of x.

$(f+g)(x) = f(x) + g(x)$

$(f-g)(x) = f(x) - g(x)$

$(f \cdot g)(x) = f(x)g(x)$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

Composition of Functions

$(f \circ g)(x) = f(g(x))$

$(g \circ f)(x) = g(f(x))$

To evaluate a composite function, work from the inside out!

The Difference Quotient

$$m = \frac{f(x+h) - f(x)}{h}$$

Examples

18. Answer the following about $f(x)$ and $g(x)$.

$$f(x) = \sqrt{x+2}, g(x) = \frac{x-3}{3x-18}$$

(a) What is $(g \circ f)(x)$?

(b) What is the domain of $(g \circ f)(x)$?

(c) What is $(g \circ f)(-4)$?

(d) What is $(g \circ f)(0)$?

Section 2.1: Quadratic Functions and Applications

Linear Functions vs. Quadratic Functions
A linear function is of the form $f(x) = ax + b$ (polynomial of degree 1) while a quadratic function is of the form $f(x) = ax^2 + bx + c$ (polynomial of degree 2)

By completing the square, the equation of the parabola $y = ax^2 + bx + c$ can always be rewritten in the form $y = a(x-h)^2 + k$ where (h, k) is the vertex of the parabola.

Examples

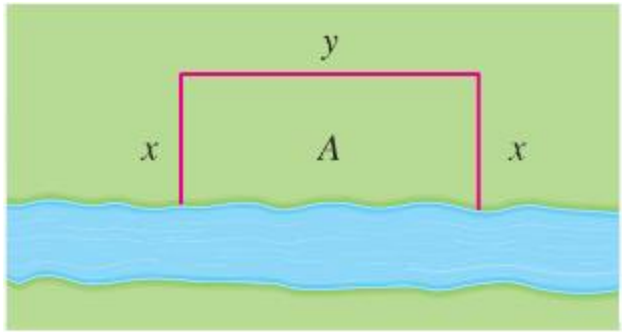
19. Solve for x.

$$\frac{x}{5x+1} + \frac{x-1}{x-2} = 1$$

20. A parabola has vertex $(1, -2)$ and passes through the point $(2,3)$.
    (a) Determine the standard equation of the parabola.

    (b) If the parabola is shifted 4 units left and 3 units up, what is the equation of the resulting parabola?

21. The zombie apocalypse is upon us. In order to save you and your friends, you need to construct an enclosure to trap them. You manage to find 100 ft of fencing and want to build a rectangular enclosure next to a river. (Hopefully, zombies can't swim)



(a) Determine the area of the enclosure as a function of x.

(b) What is the maximum area you can enclose with the amount of fence you salvaged?