

By providing my signature below I acknowledge that this is my work, and I did not get any help from anyone else:

Name (sign): \_\_\_\_\_

Name (print): \_\_\_\_\_

Student Number: \_\_\_\_\_

Instructor's Name: \_\_\_\_\_

Meeting Time: \_\_\_\_\_

Problem Number	Points Possible	Points Made
1	30	
2	10	
3	15	
4	10	
5	15	
6	10	
7	10	
8	10	
9	15	
10	15	
11	10	
12	15	
13	10	
14	25	
15	10	
16	15	
Total:	225	

- If you need extra space use the last page.
- Please show your work. **An unjustified answer may receive little or no credit.**
- Your work must be **neat**. If I can't read it (or can't find it), I can't grade it.
- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.
- Please turn off your mobile phone.
- You are only allowed to use a TI-30 calculator. No other calculators are permitted.

1. Determine **all** possible values of  $x$  that satisfy each of the relationships below.

(a) [5 pts]  $5 = e^{2x+1}$

(b) [5 pts]  $-2 = \log_{10}(1 - x^2)$

(c) [5 pts]  $2^x = 4 \cdot 5^x$

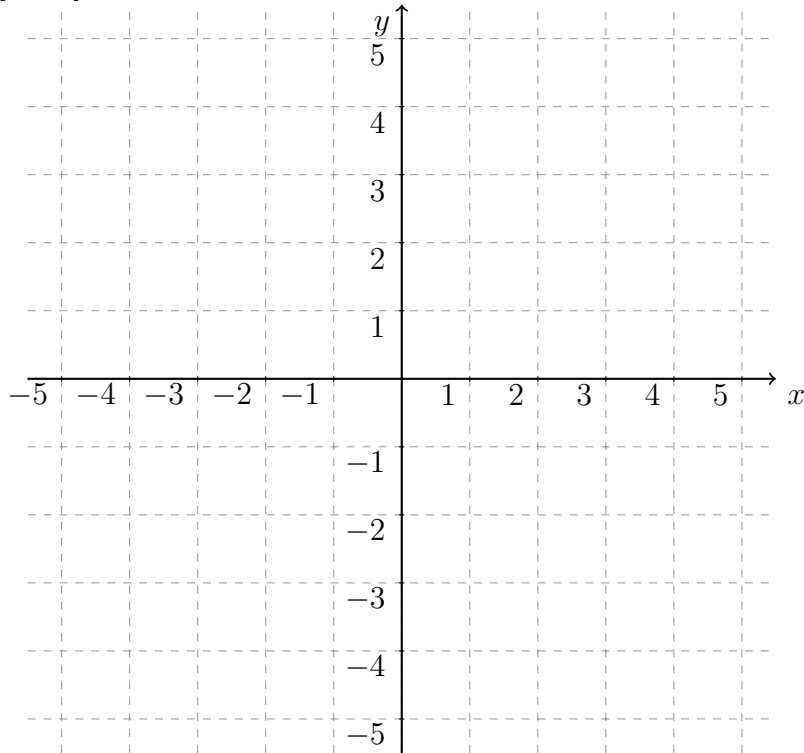
(d) [5 pts]  $\ln(x + 1) = \ln(x) + 1$

(e) [5 pts]  $\frac{1}{2} = \sin(4x + 1)$

(f) [5 pts]  $\frac{\sqrt{2}}{2} = \cos\left(\frac{x-1}{2}\right)$

2. Use the points  $P(3, -1)$ ,  $Q(-2, 1)$ , and  $R(1, -3)$  for the questions below.

(a) [5 pts] Plot and label the points  $P$ ,  $Q$ , and  $R$  using the following coordinate axes.



(b) [5 pts] Determine the distance between points  $P$  and  $Q$ .

3. Simplify each expression below to an equivalent expression that contains no trigonometric functions. Show your work, and a result from using a calculator will not receive any credit.

(a) [5 pts]  $\cos\left(\arcsin\left(\frac{1}{3}\right)\right)$

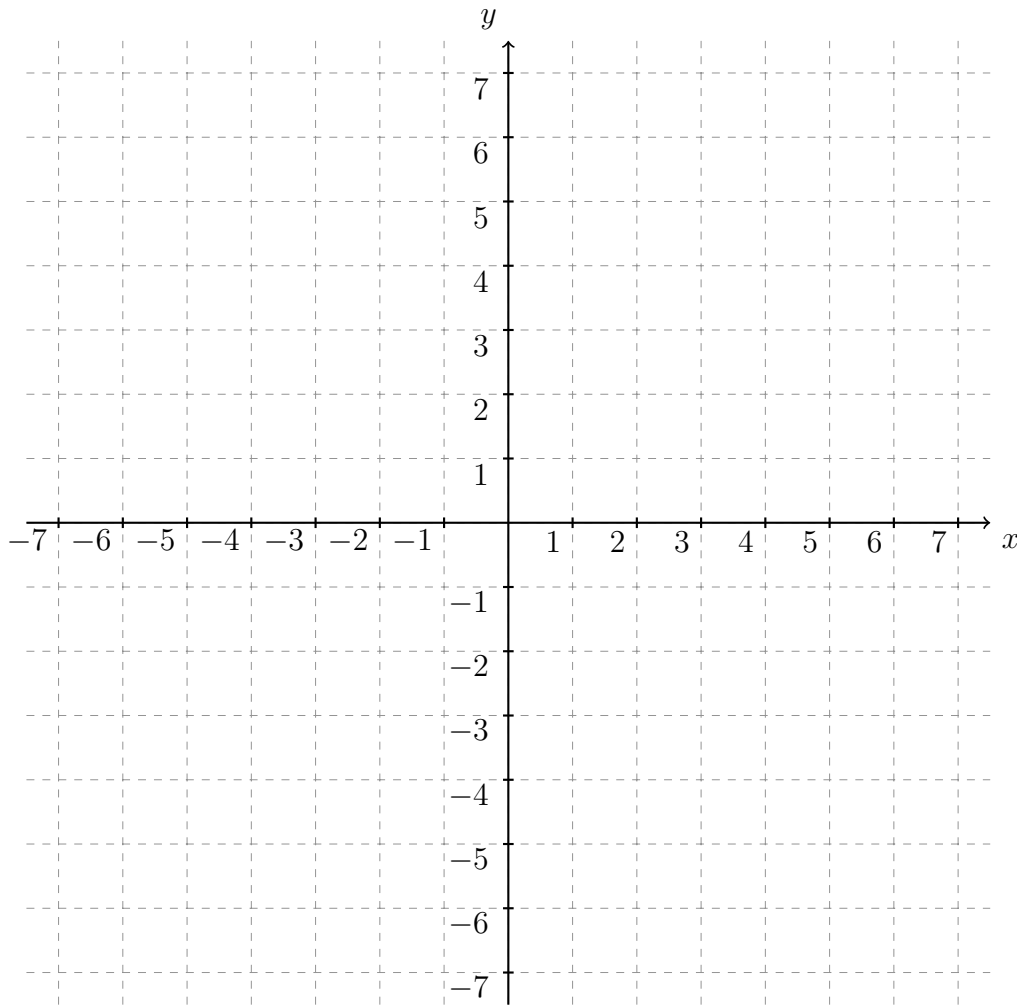
(b) [5 pts]  $\cos(\arcsin(x))$

(c) [5 pts]  $\tan(\arccos(x))$

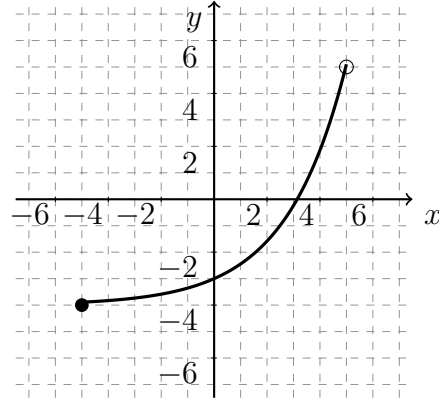
4. [10 pts] Sketch a plot of the relationship

$$x^2 + 2x + y^2 - 6y = -1$$

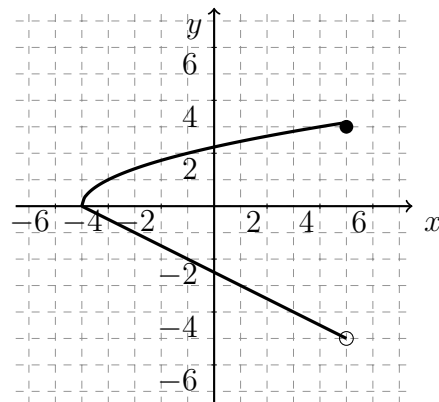
using the axes provided.



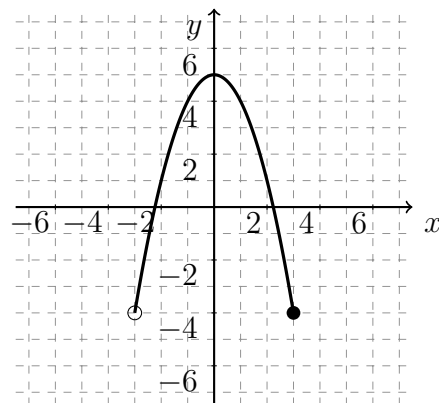
5. For each relationship below indicate which one is a function. For those that are a function provide the domain and range of the function and also indicate whether or not it is 1-1.



(a) [5 pts]



(b) [5 pts]



(c) [5 pts]



6. [10 pts] A single pulley has a diameter of 0.2 meters. A rope is put around the pulley to raise an object 4 meters straight up. Determine the angle in radians that the pulley turned. (Assume that the rope does not slip.)

7. [10 pts] A radio beacon sends out a signal that oscillates with a period of 3 seconds. The strength of the signal varies between 1 dB and 4 dB, and it has a maximum at  $t = 2$  seconds. Determine the equation for the signal assuming that it is a cosine wave.

8. The questions below refer to the function

$$g(x) = \sqrt{5-x} + 1.$$

(a) [5 pts] Determine the domain and range of the function.

(b) [5 pts] Determine the average rate of change of the function from  $x = 2$  to  $x = 4$ .

9. A train engine starts off with five-thousand gallons of fuel, and it burns the fuel at a constant rate of one-hundred and fifty gallons per hour.

(a) [5 pts] Determine the function that returns the amount of fuel remaining in the engine's tanks given the time (in hours) after it starts.

(b) [5 pts] How long can the engine go before it needs a refill.

(c) [5 pts] How do the  $x$  and  $y$ -intercepts relate to the questions asked above?

10. A quadratic function is defined to be

$$h(x) = x^2 + 8x + 10.$$

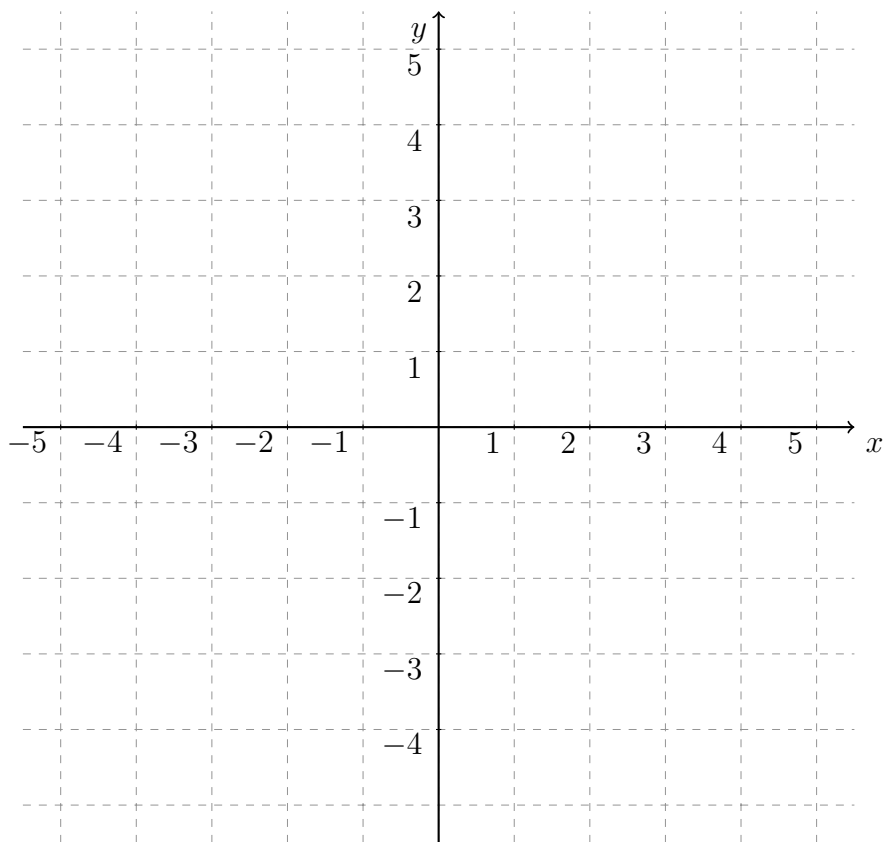
(a) [5 pts] Determine the values of  $x$  where the function is increasing.

(b) [5 pts] Determine the values of  $x$  where the function is decreasing.

(c) [5 pts] Determine the values of  $x$  where a local minimum or a local maximum occur. State whether or not the point is a local minimum or maximum.

11. [10 pts] Make a sketch of the graph of the cubic function

$$p(x) = 3x^3 - 9x^2 + 12.$$



12. [15 pts] You are browsing the web for interesting trigonometric identities, and one of your favourite pages indicates that

$$\sin\left(\frac{\pi}{12}\right) = \frac{1}{4}(\sqrt{6} - \sqrt{2}).$$

Use this information to determine the exact value of  $\sin\left(\frac{23\pi}{12}\right)$ . (Explain and provide a full justification for your answer. The number by itself or a calculator result is not worth any credit.)

13. [10 pts] A radioactive isotope has a half-life of 2,000 years. How long will it take for 80% of the material to decay?

14. The London Eye is a ferris wheel that allows observers to rotate high above the ground and obtain a spectacular view of the city. (We have a similar wheel in Atlanta the Atlanta Skyview but not nearly as large.) The height (in feet) of your "pod" above the ground on the London Eye can be modeled by the equation

$$H(t) = -197 \cos\left(\frac{\pi}{15}t\right) + 246,$$

where  $H(t)$  represents the height in feet off the ground and  $t$  represents the number of minutes elapsed since the beginning of the ride.

- (a) [5 pts] Draw a picture of the London Eye, and accurately label the height of the top and bottom of the wheel.
- (b) [5 pts] How tall is the loading platform at the bottom of the ferris wheel?
- (c) [5 pts] If you purchase a ticket for one revolution, how long is your ride in minutes?
- (d) [5 pts] What is the radius of the London Eye?
- (e) [5 pts] If we switch the units to reflect the metric system, identify the parts of the equation that would change and the parts that would remain the same. (You do not have to perform the conversation, just clearly state those that will change and those that will remain the same.)



15. [10 pts] Two chicken coops will be constructed using 120 feet of fencing. The coops will be in the shape of a rectangle with fencing across the middle dividing the rectangle into two equal rectangles. What dimensions will give the largest total area for the two coops?

16. [15 pts] A helicopter pilot spots a ship in distress. The pilot stops the helicopter, and estimates that the angle of depression to the ship is  $3^\circ$ . The pilot then lowers the helicopter 20m and estimates that the new angle of depression is  $2.5^\circ$ . How far away is the ship? (Determine the horizontal distance and not the straight line distance between the ship and helicopter.)

Extra space for work. If you want us to consider the work on this page you should write your name, instructor and meeting time below.

Name (print): \_\_\_\_\_ Instructor: Name (print): \_\_\_\_\_ Time: \_\_\_\_\_