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MATH 1101 Exam 1 Review

Spring 2018

Topics Covered

Section 2.1 Functions in the Real World

Section 2.2 Describing the Behavior of Functions

Section 2.3 Representing Functions Symbolically

Section 2.4 Mathematical Models

Section 3.1 Fundamental Concepts of Linear Functions

Section 3.2 Modeling with Linear Functions

Section 3.3 Linear Functions and Data

Section 2.1 Functions in the Real World

Functions

A function from A to B is a rule of correspondence that assigns each element of A to exactly ONE element of B. Functions are represented in one of three ways: table, graph or an equation (formula)

Domain (the inputs)

The domain of a function represents the set of all x values allowed to go into the function.

Range (the outputs)

The range of function represents the set of all y values allowed to come out of the function.

Tables

How do you determine if a table of values represents a function?

Notation: Domain and range are given in set notation using { } and are a list of values. The values in the sets are written in sequential order with no repeats.

Example 1

Do the following tables represent a function? State why or why not. If yes, give the domain and range.

x	1	5	3	2	5	7	10	12
y	6	1	3	3	2	4	8	25

x	2	4	5	6	9	11	3	1
y	1	2	2.5	3	4.5	5.5	1.5	0.5

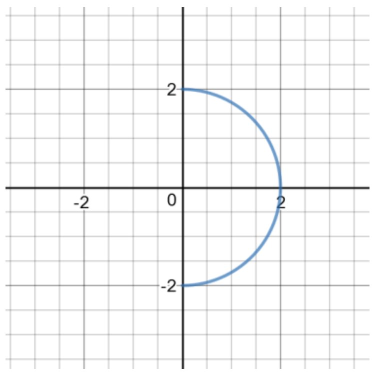
Graphs & Equations

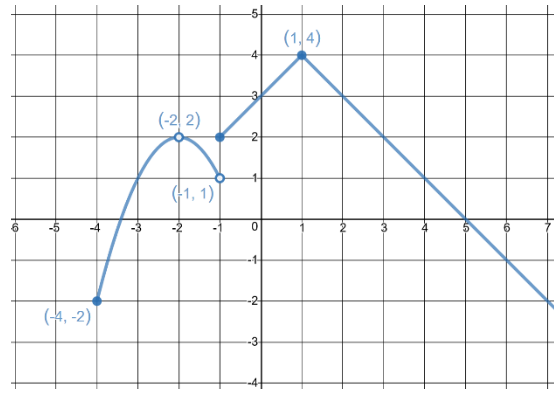
How do you determine if a table of values represent a function?

Notation: Domain and range are given in interval notation using [ ] for endpoints that *are* included in the interval and ( ) for endpoints that *are not* included in the interval. *Always* use ( ) around ∞! A combination of square brackets and parenthesis is allowed. Use ∪ (union) to connect intervals as necessary.

Example 2

Which of the following represent a relationship that is a function? State your reason for saying yes or no. If yes, give the domain and range of the function.





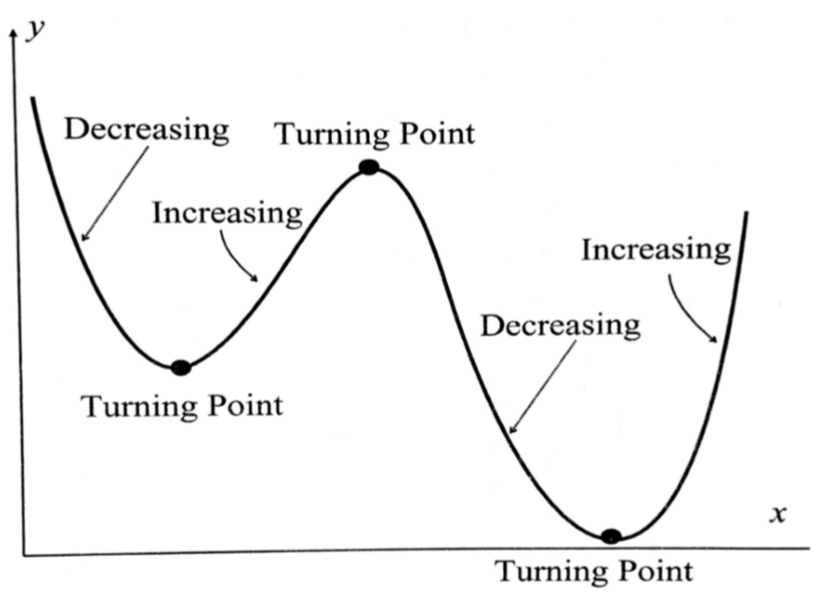
Section 2.2 Describing the Behavior of Functions

Increasing: A function is said to be increasing along an interval if the graph *rises* when read from left to right.

Decreasing: A function is said to be decreasing along an interval if the graph *falls* when read from left to right.

Constant: A function is said to be constant along an interval if the graph is *horizontal* (slope is zero) when read from left to right.

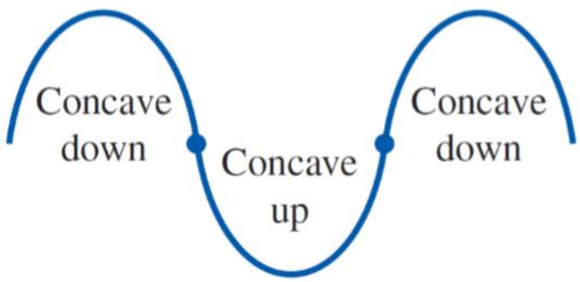
Turning Point: A function has a turning point at the location where it changes from increasing to decreasing or vice-versa. When read from left to right, the turning point is called a *local maximum* when it changes from increasing to decreasing and a *local minimum* when it changes from decreasing to increasing.

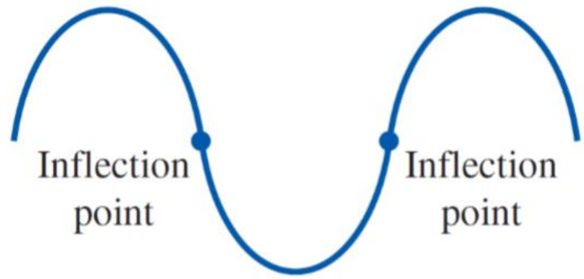


Concave Up: A function is said to be concave up if it bends upwards ('holds water') when read from left to right.

Concave Down: A function is said to be concave down if it bends downwards ('sheds water') when read from left to right.

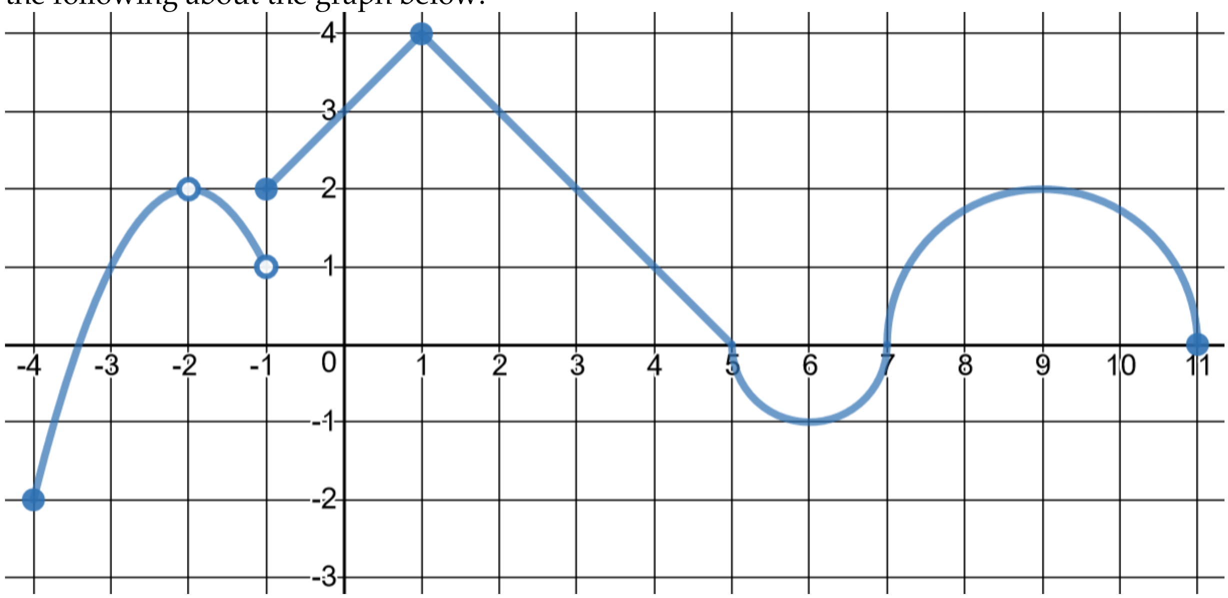
Inflection Point: A function has an inflection point at the location where it changes from concave up to concave down or vice-versa. Inflection points occur when the function is increasing or decreasing most rapidly.





Example 3

Answer the following about the graph below.



(a) Where is the graph increasing, decreasing or constant?

(b) Where is the graph concave up or concave down?

(c) Where are the locations of any local maxima, local minima or inflection points?

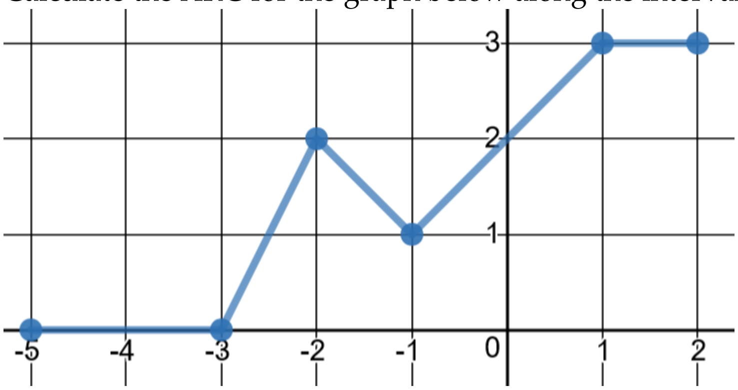
Average Rate of Change (ARC)

The average rate of change of a function on the interval $[a, b]$ is defined as the slope of the secant line that connects the two points on the graph. It can be calculated by

$$m = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

Example 4

Calculate the ARC for the graph below along the intervals $[-5, -3]$, $[-2, -1]$ and $[1,2]$.



We can use the value of the ARC to tell us information about attributes of the graph without graphing. The following table summarizes the relationship between the ARC and behavior of the graph of the function.

Interpreting a Graph Using ARC

Sign of ARC	ΔARC (from left to right)	Description of Graph
+	Increasing	Increasing, Concave Up
+	Decreasing	Increasing, Concave Down
−	Increasing	Decreasing, Concave Up
−	Decreasing	Decreasing, Concave Down

Example 5

Use the table below to describe the behavior of the function on the following intervals.

x	y	ΔARC	x	y	ΔARC
-4	-35	35	1	0	-5
-3	0	15	2	-5	5
-2	15	1	3	0	21
-1	16	-7	4	21	43
0	9	-9	5	64	----

$[-4, -2]$

$[-1, 1]$

$[-2, 5]$

Are there any local maxima or minima? What interval do they exist?

Section 2.3 Representing Functions Symbolically

When we write $y = f(x)$, it is read as "y as a function of x" where x represents the input variable and y represents the output variable. f is the name of the function. Note that variables can use any letter to represent them and are typically related to the quantity being measured. For example, t could be used to represent the variable time.

Finding the value of a function for a given x (input)

Option 1

1. Type the equation into the equation editor (Y=) where $Y_1 = f(x)$ and exit using 2ND→QUIT.
2. Go to 2ND→CALC→VALUE. Enter the x value and hit enter.

Option 2

1. Type the equation into the equation editor (Y=) where $Y_1 = f(x)$ and exit using 2ND→QUIT.
2. Go to GRAPH→TRACE. You can move the cursor along the graph to see various points.

Example 6

For the function $f(x) = 6x^2 + x - 1$, find $f(-1)$, $f(2.5)$ and $f(11)$.

Finding the x value of a function for a given y (output)

1. Type the equation into the equation editor (Y=) where $Y_1 = f(x)$ and exit using 2ND→QUIT.
2. Type the y value in Y_2
3. Go to 2ND→CALC→Intersect. Hit Enter three times.

Example 7

For what x value(s) does the function $f(x) = 6x^2 + x - 1 = 9$?

Calculating a single ARC value from a given equation for an interval $[a, b]$

Option 1

1. Type the equation into the equation editor (Y=) where $Y_1 = f(x)$ and exit using 2ND→QUIT.
2. Type $(Y_1(b) - Y_1(a))/(b - a)$. You can find Y_1 using VARS→Y-VARS→Function→ Y_1

For this method, pay close attention to your use of parenthesis!

Option 2

1. Go to the list editor STAT→Edit…
2. Set $L_1 = \{a, b\}$. The curly brackets { } can be found by 2ND→ ( )
3. Set $L_2 = Y_1(L_1)$. Use Y-VARS to grab Y_1 as described above
4. Set $L_3 = \Delta List(L_2)/\Delta List(L_1)$. You can find $\Delta List$ using 2ND→LIST→OPS→ $\Delta List$

Example 8

Find the ARC for the function $f(x) = 6x^2 + x - 1$ on the interval [2,4].

Section 2.4: Mathematical Models

A mathematical model is an equation that best fits a set of observed data points. The model CANNOT predict values outside of the domain of the original data set.

If the data set is not available and the model is given as an equation, pay attention to the context for the model. For example, do negative values make sense? Very small or large values?

Example 9
The equation $G(x) = 8.98x + 11.54$ models the expected grade on an exam based on the number of hours the student studied. Answer the following:

(a) What is the expected grade for a student who studied 5 hours?

(b) If the application domain is $[0.10]$, what is the range?

(c) How many hours should a student study to expect a grade of 95?

Section 3.1 Fundamental Concepts of Linear Functions

Linear Function
A function is linear if it can be written in the form $y = ax + b$ where a is the slope of the line and b is y-intercept. Functions where the ARC is constant are linear functions where the ARC is the slope.

Intercepts
The y-intercept (or vertical intercept) is where the line crosses the y-axis and is located at $(0, b)$. The x-intercept (or horizontal intercept) is where the line crosses the x-axis and is located at $(x, 0)$.

Example 10
Find the intercepts of the linear function $f(x) = 6x + 36$.

Example 11
Julie had $1025 is savings on June 1, 2010 when she graduated from college. Her new job will pay her $2200 per month after taxes and monthly expenses. Answer the following

(a) Find a linear function that gives Julie's net income

(b) How much money will she have made by the end of October? (Assume she is paid on the 15th of each month)

(c) How long until Julie has made $25,0000?

(d) In January, Julie's student loan payments begin. If she has a balance of $75,000 and her payments are $500 per month, find a linear function for the balance due on her loans. (Assume Julie got an interest free loan from a rich relative)

(e) How long until Julie's loan balance falls below $50,000?

Section 3.2 Modeling with Linear Functions

Working with two linear models

If two linear models are using the same input and output variables, we can find where the models are equal by looking at their intersection.

Example 12

Two different student groups at UGA are fundraising for Relay For Life. Group A's fundraiser is modeled by $f(t) = 255t + 322$ and group B's fundraiser is modeled by $g(t) = 145t + 536$ where t is measured in weeks. If the two groups start their fundraiser at the same time, how long until they have raised the same amount?

Example 13

Bill just got a new sales job. He can take a base salary of $10,000 per year plus 20% of his sales revenue or he can take a base salary of $50,00 plus a 5% of his sales revenue.

(a) Find the linear models for his annual salary for each of the two options.

(b) How much will Bill earn under each plan if he sells $40,000 worth of product?

(c) How much should Bill sell to make more money off the first option? How much would he make that year?

Piecewise Functions

Functions whose ARC only stays constant for different intervals of the input value. We "piece" portions of different linear functions together.

Example 14

Christy, Tim and Dory are taking a road trip to see their favorite band Family and Friends. They leave Athens at noon and drive for 4 hours at 60mph, they stop for lunch for an hour, get back on the road and drive 50mph for 2 more hours and when they hit town there is a traffic jam so only go 20mph for the remaining half hour.

(a) Sketch a function of distance vs. time for their journey.

(b) What is the formula for the distance function?

(c) What time do they arrive?

(d) How far was their total drive?

(e) How far had they traveled by 5:30PM?

Section 3.3 Linear Functions and Data

How do we determine if a function is linear from looking at a table of data points?
You could plot the data points but looks can be deceiving. You need to verify that the function is truly linear! A linear function will have a constant ARC.

Example 15
Are the two following functions linear? If yes, give the linear model.

x	5	10	15	20	25
y	2.46	3.81	6.43	7.78	9.19

x	5	10	15	20	25
y	5.53	8.04	10.55	13.06	15.57

How do we work with functions that are 'almost' linear?
If the data points show that the function behaves linearly but doesn't have a constant ARC, we can use Linear Regression to find the line of best fit that will approximate a model for us. We'll get into that next time…

Useful Calculator Screenshots

Calculator Tip #6

A function $y = f(x)$ can be evaluated anywhere on its domain.

- Store the function as **Y1** (see tip #4)
- Press **2ND** · **Quit**
- Press **VARS** · **YVARS** · **Function** · **Y1** · **(**
- Type the desired x-value
- Press **)** · **ENTER**

Try it: Store $f(x) = 430 \times 1.00125^{12x}$ as **Y1**. Use **Y1** to find $f(10)$.

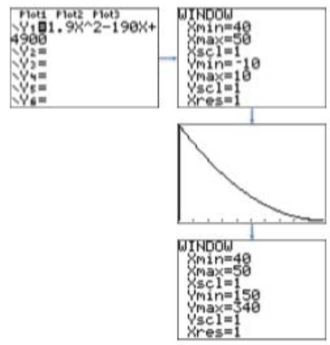
Y1 is located at **VARS** · **Y-VARS** · **Function**

Calculator Tip #8

To graph a function $y = f(x)$ on the interval $[a, b]$:

- Store the function as **Y1** (see tip #4)
- Press **WINDOW**
- Set **Xmin** = the interval's left-hand endpoint a
- Set **Xmax** = the interval's right-hand endpoint b
- Press **ZOOM**
- Select **ZoomFit**

Try it: Graph $f(x) = 1.9x^2 - 190x + 4900$ on the interval [40, 50].



Set **Xmin** and **Xmax** only, then run **ZoomFit**

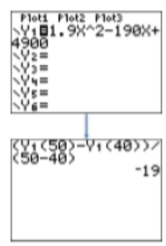
Notice how **ZoomFit** set **Ymin** and **Ymax**

Calculator Tip #10

To calculate the average rate of change of a function $y = f(x)$ on the interval $[a, b]$:

- Store the function as **Y1** (see tip #4)
- Press **2ND** · **Quit**
- Type (**Y1**(b)−**Y1**(a))/(b−a)
- Press **Enter**

Try it: Calculate the average rate of change of $f(x) = 1.9x^2 - 190x + 4900$ on the interval [40, 50].



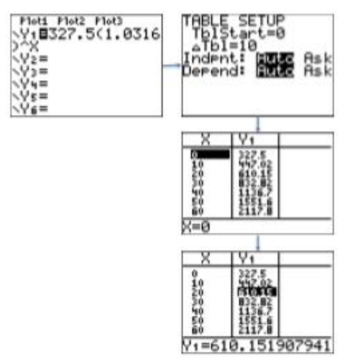
Y1 is located at **VARS** · **Y-VARS** · **Function**

Calculator Tip #11

To generate a table of function values for $y = f(x)$ for exploring the function's behavior:

- Store the function as **Y1** (see tip #4)
- Press **2ND** · **TBLSET**
- Set **TblStart** = the desired starting x-value
- Set **ΔTbl** = the desired change in x each time
- Set **Indpnt** = Auto
- Set **Depend** = Auto
- Press **2ND** · **TABLE** (cursor up and down as needed)

Try it: For $f(x) = 327.5(1.0316)^x$, generate a table of values for $x = 0, 10, 20, 30, \ldots$



To see more function values, cursor up or down

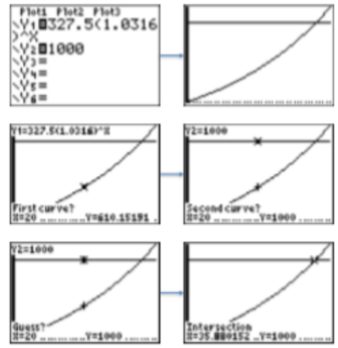
Cursor onto a function value to see more decimal places

Calculator Tip #12

To solve the equation $f(x) = g(x)$, assuming there's a solution on the interval $[a, b]$:

- Store $f(x)$ as **Y1** (see tip #4)
- Store $g(x)$ as **Y2** (see tip #4)
- Graph the functions on $[a, b]$ (see tip #8)
- Press **2ND** · **CALC** · **intersect**
- Press **ENTER** to select **Y1** for "First curve?"
- Press **ENTER** to select **Y2** for "Second curve?"
- Press **ENTER** to select the current cursor position for "Guess?"

Try it: Solve $327.5(1.0316)^x = 1000$. From the table of function values on the previous slide, it follows that there's a solution on the interval [0, 40].



Set **Xmin** = 0, and **Xmax** = 40 only, then run **ZoomFit**

With cursor on **Y1**, press **ENTER**; With cursor on **Y2**, press **ENTER**

Press **ENTER** one more time; $f(35.88) \approx 1000$

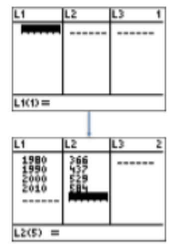
Calculator Tip #14

A set of data points $(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots$ can be stored in the calculator.

- Store $x_1, x_2, x_3, \ldots$ in list **L1** (see tip #13)
- Store $y_1, y_2, y_3, \ldots$ in list **L2** (see tip #13)

Try it: Store the following data points in the calculator.

Year t	1980	1990	2000	2010
Population P (thousands)	366	437	529	584



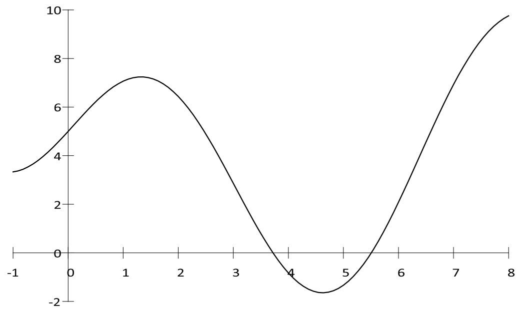
In **STAT** · **Edit** with **L1** and **L2** ready for input

Additional Practice Problems

1. Is x a function of y, is y a function of x?

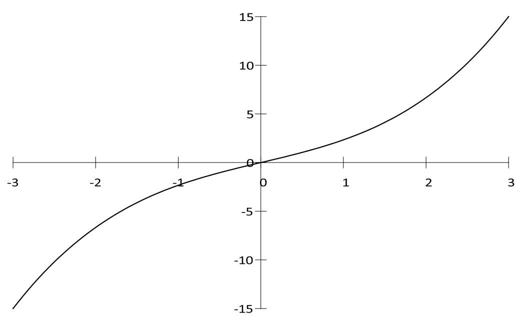
x	1	2	3	4	5	6
y	3	1	3	2	5	0

2. Is x a function of y, is y a function of x?



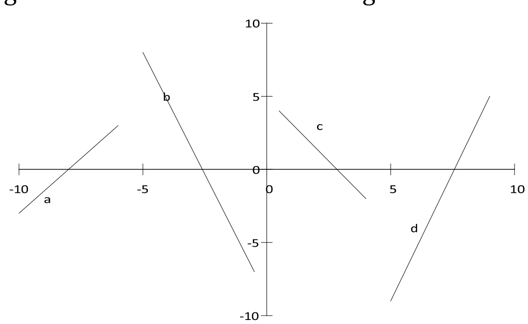
3. Let N be the value of the NASDAQ stock market at the end of each day d.Is N a function of d?______ Is d a function of N? ______

4. Is x a function of y, is y a function of x?



In the the interval (1,3), the function is [increasing or decreasing] at an [increasing or decreasing] rate.
In the the interval $(-3,-1)$, the function is [increasing or decreasing] at an [increasing or decreasing] rate.

5. Rank the slopes of the line segments below from lowest to highest.

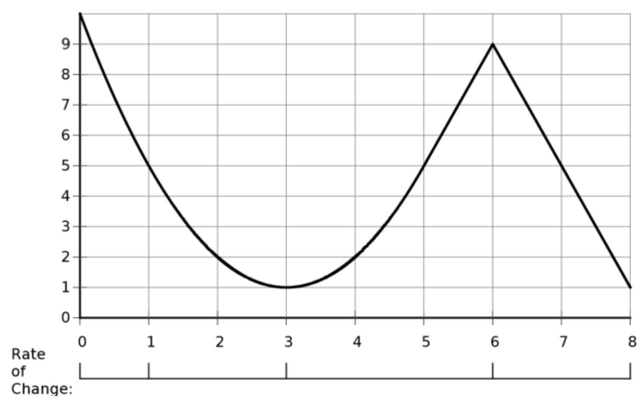


1. ______ 2. ______ 3. ______ 4. ______

6. Find the domain and range of:

x	1	2	3	4	5	6
y	3	1	3	2	5	0

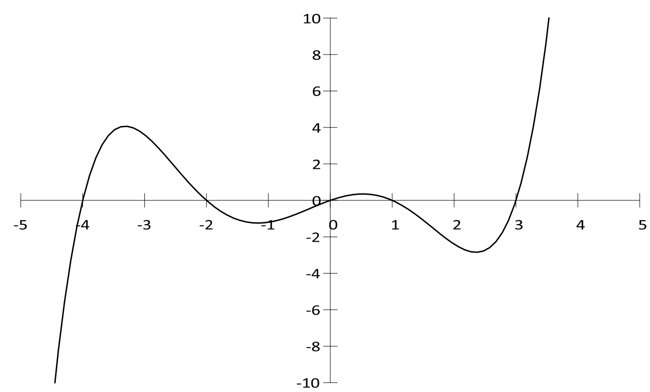
7. Find the average rate of change on the intervals in the image.



(a) $[0,1]$____________
(b) $[1,3]$____________
(c) $[3,6]$____________
(d) $[6,8]$____________
(e) $[5,7]$____________

(f) Without calculating anything more: [7,8]____________

8. Use this graph to answer the parts below:



(a) Is this a function?
(b) What is the domain? What is the range?
(c) How many turning points does it have? How many inflection points?
(d) Is there a local maximum? Is there more than one? What are they?
(e) The equation $y = -1$ has how many solutions?
(f) The equation $y = 3$ has how many solutions?
(g) Does the function have an inverse?

9. Your lemonade stand on North Campus sold 59 cups when your price was \$.50 per cup the next day you changed the price to \$.75 and sold 44 cups.

    (a) Assume the function between sales and price is linear, write a function $s(p)$ where s is the number of cups sold and p is the price charged.

    (b) How many cups do you sell if you charge \$1.00?

    (c) You only brought 20 cups, you want to set your price so you sell all of them, what should your price be?(d) How much money do you make selling those 20 cups for the price you found?

10. The population of Makebelievia is $p(t) = -13t^2 + 156t + 668$ where t is years after January 1, 2000. This equation works from Jan 1, 2000 to Jan 1, 2014.

    (a) What is the domain in terms of t?

    (b) What is the range?

    (c) How many turning points does it have? How many inflection points?

    (d) Is this concave up or concave down, or first one then the other?

    (e) Estimate the maximum population of Makebelievia.

    (f) Complete the table:

t	$p(t)$	ARC
0	______	______
2	______	______
4	______	______
5	______	______
8	______	______
10	______	______
12	______	______
14	______	Blank

11. You are heading to the Georgia vs. Florida game! You begin your trip traveling 60mph for 3 hours. You then decide to stop for lunch for an hour. After lunch you continue your trip driving 55mph for 2 hours. Once you enter Jacksonville you hit heavy traffic before you reach the stadium. Therefore you travel 20mph for half an hour until you are able to park. GO DAWGS! Let $d(t)$ represent the total number of miles that you have traveled and t represent the number of hours.

    (a) Write a formula for your distance traveled in terms of number of hours.

    (b) What is the domain and range of your function?

    (c) How far have you traveled after five and a half hours

    (d) When are you exactly 100 miles from home?

12. Georgia Power charges different rates depending on how much power a customer uses. The amount charged ismodeled by $f(x)$ where x is the number of kWh used.

$$f(x) = \begin{cases} 15 + .15x & 0 \le x \le 100 \\ 5 + .25x & 100 < x \le 2000 \\ 305 + .10x & 2000 < x \end{cases}$$

    (a) What is your power bill if you use 80 kWh?

    (b) What is your power bill if your roommate leaves their space heater on high over the break and you use280 kWh?

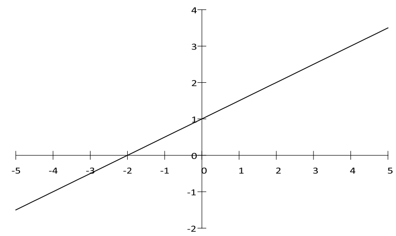
    (c) What is your power bill if you are running an industrial aluminum refinery and you use 6000 kWh?

(d) if your power bill is \$103, how much electricity did you use?(e) What is the average rate of change over the interval [10,50] (f) In English, what does $f(x) = 15 + .15x$ mean?

13. Is this a linear function of x? if yes, what is the function?

x	0	1	2	3	4	5
y	3.500	.350	-.954	-1.956	-2.80	-3.54

14. Is this a linear function of x? If yes, what is the function?



15. Is this a linear function of x? if yes, what is the function?

x	0	1	2	3	4	5
y	7.100	3.950	.800	-2.350	-5.500	-8.650

16. Makebelievia Inc. is making automatic homework machines, they can make 2000 the first year, increasing by250 every year, 2750 people want to buy them the first year, increasing by 100 every year. What's the first year that everyone who wants one will be able to buy one? (number made ≥ number wanted)

Answers:

1. x is not a function of y, y is a function of x
2. x is not a function of y, y is a function of x
3. N is a function of d, d is not a function of N
4. x is a function of y, y is a function of x
   In the the interval (1,3), the function is [**increasing** or decreasing] at an [**increasing** or decreasing] rate.
   In the the interval $(-3,-1)$, the function is [**increasing** or decreasing] at an [increasing or **decreasing**] rate.
5. b, c, a, d
6. Domain: {1,2,3,4,5,6} Range: {0, 1, 2, 3, 5}
7. For the intervals,
   (a) $[0,1] = -5$
   (b) $[1,3] = -2$
   (c) $[3,6] = 8/3$
   (d) $[6,8] = -4$
   (e) $[5,7] = 0$
   (f) $[7,8] = -4$
8. (a) yes
   (b) Domain: $(-\infty,\infty)$ Range $(-\infty,\infty)$
   (c) turning points 4, inflection points 3
   (d) 2 local maximums, (-3.3,4), (.5,.5)
   (e) 5
   (f) 3
   (g) no
9. (a) $s(p) = -60p + 89$
   (b) 29
   (c) $1.15
   (d) $23
10. (a) [0,14]
   (b) [304,1136]
   (c) 1 turning point, but not in the domain. No inflection points.
   (d) Concave down
   (e) 668

(f)

t	$p(t)$	ARC
0	668	130
2	928	78
4	1084	26
5	1136	−26
8	1084	−78
10	928	−130
12	668	−182
14	304	Blank

11. (a)

$$d(t) = \begin{cases} 60t + 0, 0 \leq t \leq 3 \\ 0t + 180, 3 < t \leq 4 \\ 55t - 40, 4 < t \leq 6 \\ 20t + 710, 6 < t \leq 6.5 \end{cases}$$

(b) Domain: [0,6.5] Range: [0,720]

(c) $d(5.5) = 262.5$

(d) $t = 1.67$, 1:40

12. (a) $27

(b) $75

(c) $905

(d) 392 kWh

(e) .15

(f) There is a 15 dollar monthly charge, and every kWh costs $.15.

13. No.

14. Yes.

15. Yes.

16. $t = 5$.