By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (sign): _____ Name (print): _____

Student Number: _____

Instructor's Name: _____ Class Time: _____

Problem Number	Points Possible	Points Made
1	25	
2	20	
3	10	
4	10	
5	15	
6	5	
7	10	
8	10	
9	20	
10	15	
11	10	
12	10	
13	10	
Total:	170	

- If you need extra space use the last page.
- Please show your work. **An unjustified answer may receive little or no credit.**
- If you make use of a theorem to justify a conclusion then state the theorem used by name.
- Your work must be **neat**. If I can't read it (or can't find it), I can't grade it.
- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.
- Please turn off your mobile phone.
- You are only allowed to use a TI-30 calculator. No other calculators are permitted.
- A calculator is not necessary, but numerical answers should be given in a form that can be directly entered into a calculator.

1. Determine the first derivative of each of the following functions. Print your answer in the box provided.

_____ (a) [5 pts] $f(x) = e^{3x} + 5x^2 + 2001$.

$f'(x) =$

_____ (b) [5 pts] $h(x) = x^2 \cos(x) + 42$.

$h'(x) =$

_____ (c) [5 pts] $G(r) = \dfrac{r+4}{3r^2+7}$.

$G'(r) =$

(d) [5 pts] $M(x) = \arctan(3x)$.

_____

$M'(x) =$

_____ (e) [5 pts] $Q(s) = (1+s)^{\frac{1}{s}}$.

$Q'(s) =$

2. Determine **the most general anti-derivative** indicated in each of the following expressions. Print your answer in the box provided.

——— (a) [5 pts] $\int (5x + 3)\ dx.$

Anti-Derivative:

——— (b) [5 pts] $\int \left(\cos(4x) + e^{2x} + 9\right)\ dx.$

Anti-Derivative:

(c) [5 pts] $\int \frac{x}{3-x^2}\, dx.$

Anti-Derivative:

(d) [5 pts] $\int \sin(x)\cos(x)\, dx.$

Anti-Derivative:

3. Determine the value of each of the following limits. Indicate if a limit approaches ∞ or $-\infty$ otherwise print DNE if the limit does not exist. **Show all of your work and justify your conclusions.**

(a) [5 pts] $\lim_{x\to 0} \dfrac{\sin^2(x)}{1-\cos(x)}$.

(b) [5 pts] $\lim_{x\to 1} f(x)$ where

$$f(x) = \begin{cases} x+2 & x \le 1, \\ 1 & x > 1. \end{cases}$$

4. Use the following tables to answer each of the questions below.

x	1	2	3	4	5	6	7	8
$f(x)$	4	3	5	8	6	7	1	2
$f'(x)$	3	2	1	6	4	5	8	7

x	1	2	3	4	5	6	7	8
$g(x)$	6	5	1	2	8	7	4	3
$g'(x)$	1	4	2	9	3	6	8	5

_____ (a) [5 pts] Determine the value of $p'(2)$ where $p(x) = f(x) \cdot g(x)$.

$p'(2) =$

_____ (b) [5 pts] Determine the equation for the tangent line of $p(x) = f(x) \cdot g(x)$ at $x = 2$.

Tangent Line:

5. The following questions refer to the limit definition of the derivative.

(a) [5 pts] State the limit definition of the derivative of a function, $f(x)$.

_____

(b) [10 pts] Use the **limit definition of the derivative** to show that

$$\frac{d}{dx}\left(\frac{x}{x+1}\right) = \frac{1}{(x+1)^2}$$

6. [5 pts] Evaluate

$$\frac{d}{dx}\int_0^{x^2} \frac{\sin(t)}{1+t}\, dt.$$

7. The efficiency of a Carnot heat engine is defined to be

$$E(T) \;=\; 1 - \frac{C}{T},$$

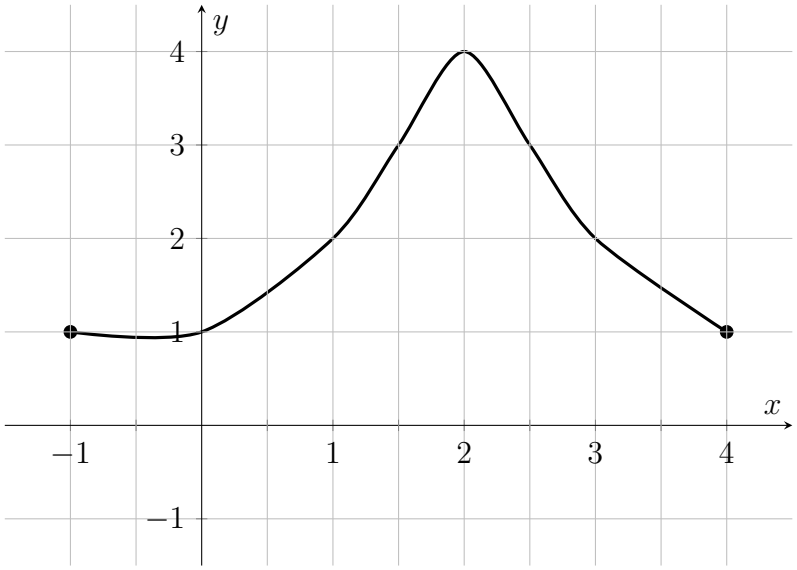
where C is the temperature of the surrounding environment in Kelvin, and T is the temperature of the heat source. For the following questions assume that $C = 300$ Kelvin is a constant.

_____ (a) [5 pts] Determine the linearization of $E(T)$ at $T = 500$K.

(b) [5 pts] Use the linearization to approximate the change in E if $T = 500 \pm 20$K.

_____

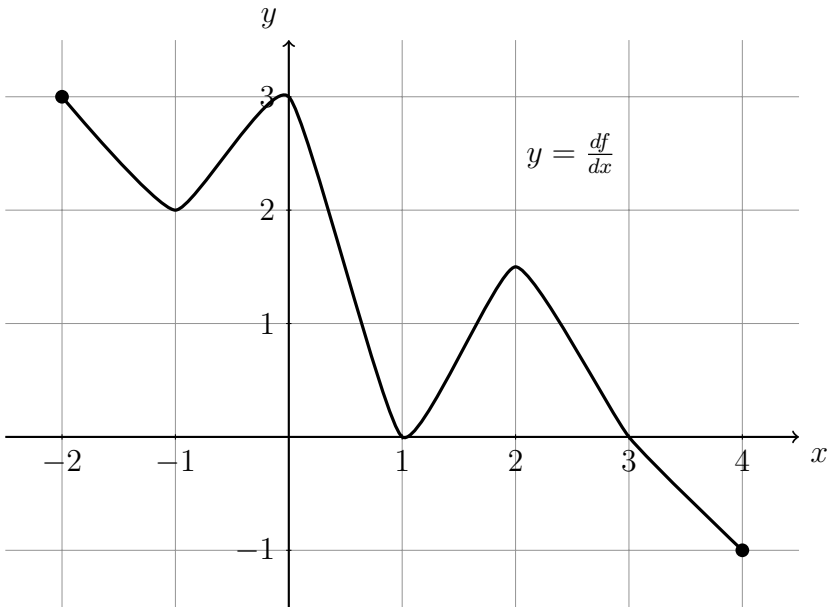
8. The graph of a function is given in the plot below. The domain of the function is the closed interval $[-1, 4]$. Use the plot to answer each of the following questions.



_____ (a) [5 pts] Estimate the area under the curve from $x = 1$ to $x = 3$ using a Riemann sum. The Riemann sum should use four rectangles of equal width, and use the left endpoint of each subinterval.

_____ (b) [5 pts] If you use the left endpoints on each subinterval to construct a Riemann sum to estimate the area from $x = 2$ to $x = 4$ will your estimate be more than, equal to, or less than the true value?

9. The graph of the derivative, $f'(x)$, of a function is given in the plot below, and the domain of the derivative is $[-2, 4]$. Answer each of the following questions.



_____ (a) [5 pts] For what values of x, if any, are there local extrema for f? (Indicate which values give local maxima and which give local minima.) Ignore the endpoints, $x = -2$ and $x = 4$.

_____ (b) [5 pts] For what intervals of x, if any, is f increasing?

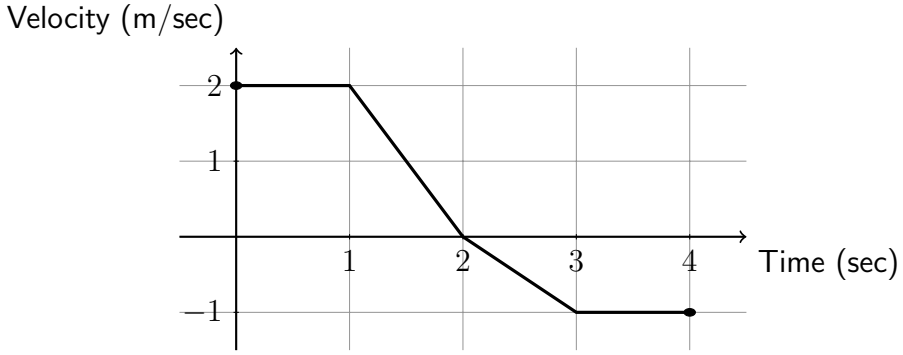
(c) [5 pts] For what values of x, if any, does f have an inflection point.

_____

(d) [5 pts] For what intervals of x, if any, is f concave down?

_____

10. An object is moving on a straight, level track. The graph of its velocity in meters per second as a function of time in seconds is shown in the plot below. Answer each of the following questions.



_____ (a) [5 pts] The object's start point is its position at $t = 0$ seconds. How far is the object from its start point at the time $t = 4$ seconds?

_____ (b) [5 pts] Determine the average velocity of the object between $t = 0$ seconds and $t = 4$ seconds.

_____ (c) [5 pts] What is the total distance traveled from $t = 0$ seconds to $t = 4$ seconds?

11. [10 pts] A car is moving North at 65 miles per hour. A person is walking due East on a different road. Determine how fast the person is moving at the moment when the person is 50 miles West and 70 miles South of the car and the distance between the person and the car is increasing at a rate of 55 miles per hour.

12. [10 pts] A component of an engine will be connected to a heat sink by a cylindrical rod of length 8 cm with radius r. The rate of heat flow through the rod is given by

$$G = \frac{1}{8}\mu r^2,$$

where $\mu > 0$ is the *thermal conduction coefficient*. The cost to make the rod is equal to the radius plus the thermal conduction coefficient. If \$30 is allocated to produce the rod, determine the radius, r, and thermal conduction coefficient, μ, of the rod that will maximize the rate of heat flow.

13. [10 pts] Find all points, other than (0,0), where the curve

$$x^3 + xy + y^3 = 0$$

has a vertical tangent line. *Your answer(s) should be in the form of coordinate pairs.*

Extra space for work. **Do not detach this page.** If you want us to consider the work on this page you should print your name, instructor and class meeting time below.

Name (print): _____ Instructor (print): _____ Time: _____