By providing my signature below I acknowledge that I abide by the University’s academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (sign):  
Name (print):  
Student Number:  
Instructor’s Name:  
Class Time:  

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<td>272</td>
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- If you need extra space use the last page. Do not tear off the last page!
- Please show your work. An unjustified answer may receive little or no credit.
- If you make use of a theorem to justify a conclusion then state the theorem used by name.
- Your work must be neat. If I can’t read it (or can’t find it), I can’t grade it.
- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.
- Please turn off your mobile phone.
- You are only allowed to use a TI-30XS Multiview calculator. No other calculators are permitted, and sharing of calculators is not permitted.
- A calculator is not necessary, but numerical answers should be given in a form that can be directly entered into a calculator.
1. Determine the following limits. If you answer with $\infty$ or $-\infty$, briefly explain your thinking. Print your final answer in the box provided.

(a) [5 pts] \[ \lim_{x \to 2} (3x^2 + 7x - 5) \]
\[ = 3 \cdot 2^2 + 7 \cdot 2 - 5 \]
\[ = 12 + 14 - 5 \]
\[ = 21 - 5 \]
answer: 21

(b) [5 pts] \[ \lim_{x \to 1^-} \frac{2x}{x - 1} \]
\[ 2x \to 2 \text{ as } x \to 1^- \]
\[ x - 1 \to 0 \text{ and is negative as } x \to 1^- \]
answer: $-\infty$

(c) [8 pts] \[ \lim_{x \to \infty} \frac{\ln(5x)}{x^3 + 1} \]
\[ = \lim_{x \to \infty} \frac{\ln(5)}{3x^2} \]
\[ = \lim_{x \to \infty} \frac{1}{3x^2} \]
\[ = 0 \]
answer: 0
2. Determine the first derivative of each of the following functions. Print your answer in the box provided. **You do not have to simplify your answers or explain your steps.**

(a) [4 pts] \( f(x) = 8x^3 - 15x + 12 \)

\[
 f'(x) = 24x^2 - 15
\]

(b) [6 pts] \( g(t) = \frac{\sin(t)}{t} \)

\[
g'(t) = \frac{t \cos(t) - \sin(t)}{t^2}
\]

(c) [6 pts] \( f(x) = \frac{e^x}{2x + 1} \)

\[
f'(x) = \frac{(2x+1)e^x - e^x \cdot 2}{(2x+1)^2} = \frac{e^x(2x-1)}{(2x+1)^2}
\]

(d) [10 pts] \( h(x) = (4x - 3)^2 \arctan(x) \)

\[
h'(x) = (4x-3)^2 \frac{1}{1+x^2} + \arctan(x) \cdot 2(4x-3) \cdot 4
\]

\[
h'(x) = \frac{(4x-3)^2}{1+x^2} + 8(4x-3) \arctan(x)
\]
3. (a) [8 pts] Determine \( \frac{dy}{dx} \) for the equation \( y^3 - x^4y = 6 \). Print your answer in the box provided. You do not have to simplify your answer.

\[
3y^2 \frac{dy}{dx} - (x^4 \frac{dy}{dx} + 4x^3y) = 0
\]

\[
3y^2 \frac{dy}{dx} - x^4 \frac{dy}{dx} - 4x^3y = 0
\]

\[
3y^2 \frac{dy}{dx} - x^4 \frac{dy}{dx} = 4x^3y
\]

\[
\frac{dy}{dx} (3y^2 - x^4) = 4x^3y
\]

\[
\frac{dy}{dx} = \frac{4x^3y}{3y^2 - x^4}
\]

(b) [8 pts] Determine an equation of the tangent line to the curve \( y^3 - x^4y = 6 \) at the point \((1, 2)\).

\[
\left. \frac{dy}{dx} \right|_{(1,2)} = \left. \frac{4 \cdot 1 \cdot 2}{3 \cdot 4 - 1} \right| = \frac{8}{11}
\]

Equation:

\[
y - 2 = \frac{8}{11} (x - 1)
\]
4. Determine the following indefinite integrals. Print your answer to each part in the box provided.

(a) [4 pts] \[ \int (-4x^7 + 8x^5 + 12) \, dx \]

\[
= -\frac{4}{8} x^8 + \frac{8}{6} x^6 + 12x + C
\]

Final answer:

\[
-\frac{1}{2} x^8 + \frac{4}{3} x^6 + 12x + C
\]

(b) [6 pts] \[ \int \left( \sec^2(t) + \frac{1}{t} \right) \, dt \]

Final answer:

\[
\tan(t) + \ln|t| + C
\]

(c) [10 pts] \[ \int \frac{x^4}{\sqrt{x^5 + 3}} \, dx \]

\[
= \int (x^5 + 3)^{-\frac{1}{2}} \cdot x^4 \, dx
\]

\[
= \frac{1}{5} \int (x^5 + 3)^{-\frac{1}{2}} \cdot 5x^4 \, dx = \int u^{-\frac{1}{2}} \, du = \frac{2}{5} u^{\frac{1}{2}} + C
\]

Final answer:

\[
\frac{2}{5} \sqrt{x^5 + 3} + C
\]
5. Evaluate the following definite integrals. Print your answer in the box provided.

(a) \[ \int_{1}^{8} \left( x^{2/3} - \frac{1}{x^{4/3}} \right) \, dx \]
\[
= \int_{1}^{8} \left( x^{2/3} - x^{-4/3} \right) \, dx \\
= \left[ \frac{3}{5} x^{5/3} + 3 x^{-1/3} \right]_{1}^{8} \\
= \frac{3}{5} (8)^{5/3} + 3 \cdot 8^{-1/3} - \left( \frac{3}{5} + 3 \right) \\
\Rightarrow \quad \text{Value: } \frac{184}{10} - \frac{15}{10} = \frac{171}{10} = 17.1
\]

(b) \[ \int_{0}^{1/2} \frac{-1}{\sqrt{1 - x^2}} \, dx \]
\[
= \left[ \arccos(x) \right]_{0}^{\sqrt{1/2}} \\
= \arccos\left( \frac{\sqrt{2}}{2} \right) - \arccos(0) \\
= \frac{\pi}{3} - \frac{\pi}{2} \\
\Rightarrow \quad \text{Value: } -\frac{\pi}{6}
\]

(c) \[ \int_{0}^{\pi/4} \sin(4x)e^{\cos(4x)} \, dx \]
\[
u = \cos(4x) \\
du = -4\sin(4x) \, dx \\
\begin{align*} 
\int \frac{1}{4} e^u \, du & = \left[ -\frac{1}{4} e^u \right]_{1}^{-1} \\
\int_{0}^{\pi/4} \sin(4x) & = \frac{-1}{4} e^{-1} - \left( -\frac{1}{4} e^1 \right) \\
& = -\frac{1}{4} e^{-1} + \frac{e}{4} \\
\end{align*}
\]
6. (a) [5 pts] State the limit definition of the derivative of $f(x)$.

\[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \text{or} \quad f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a} \]

(b) [10 pts] Use the limit definition of the derivative to show that the derivative of $f(x) = 12x - 2x^2$ is $f'(x) = 12 - 4x$. (You will receive 0 points for using the power rule.)

\[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]
\[ = \lim_{h \to 0} \frac{12(x+h) - 2(x+h)^2 - 12x + 2x^2}{h} \]
\[ = \lim_{h \to 0} \frac{12x + 12h - 2(x^2 + 2xh + h^2) - 12x + 2x^2}{h} \]
\[ = \lim_{h \to 0} \frac{12x + 12h - 2x^2 - 4hx - 2h^2 - 12x + 2x^2}{h} \]
\[ = \lim_{h \to 0} \frac{12h - 4hx - 2h^2}{h} \]
\[ = \lim_{h \to 0} (12 - 4x - 2h) \]
\[ = 12 - 4x \]

(c) [5 pts] Determine all values of $x$ for which the graph of $f(x) = 12x - 2x^2$ has a horizontal tangent line.

\[
\begin{align*}
12 - 4x &= 0 \\
12 &= 4x \\
3 &= x
\end{align*}
\]
7. [15 pts] Determine the absolute maximum and absolute minimum values of \( f(x) = 2x\sqrt{9-x} \) on the interval \([-1, 9]\).

\[
\begin{align*}
f'(x) &= 2x \cdot \frac{1}{2}(9-x)^{-\frac{1}{2}} + \sqrt{9-x} \cdot 2 \\
&= \frac{-x}{\sqrt{9-x}} + \frac{2\sqrt{9-x}}{1} \cdot \frac{\sqrt{9-x}}{\sqrt{9-x}} \\
&= \frac{-x + 2(9-x)}{\sqrt{9-x}} \\
&= \frac{-x + 18 - 2x}{\sqrt{9-x}} \\
&= \frac{18 - 3x}{\sqrt{9-x}}
\end{align*}
\]

\( f' \) are not critical numbers (domain)

\( f'(x) = 0 : \quad x = 6 \)

\[
\begin{array}{c|c|c}
X & f(x) & \\
-1 & -2\sqrt{10} & \text{absolute min is } -2\sqrt{10} \\
6 & 12\sqrt{3} & \text{absolute max is } 12\sqrt{3} \\
9 & 0 & \\
\end{array}
\]
8. The graph below is the graph of the derivative of $f(x)$. Use it to answer the questions that follow. The grid lines are one unit apart, and the domain of $f$ is $(0, 7)$.

\[\frac{df}{dx}\]

(a) [5 pts] Determine all critical numbers (critical points) of $f$.

$x = 1, 3, 6$

(b) [5 pts] Determine the intervals on which $f$ is increasing.

\[ (0, 1] \cup [6, 7) \]

(also accepted: $[0, 1) \cup (6, 7)$)

(c) [5 pts] Determine all values of $x$ at which $f$ has a local minimum.

$f'$ changes sign from $-$ to $+$ at $x = 6$

(d) [5 pts] Determine the intervals on which $f$ is concave up.

$f$ is concave up where $f'$ is increasing:

\[ [2, 3] \cup [5, 7] \]

(also accepted: $[2, 3) \cup (5, 7)$)
9. For this problem, use \( f(x) = 3x^2 + 4 \) on the interval \([0, 2]\). Its graph is provided to the right.

(a) [5 pts] Determine a Riemann sum for \( f \) on the interval \([0, 2]\) using 3 subintervals of equal width and using right endpoints on each subinterval.

\[
\Delta x = \frac{2-0}{3} = \frac{2}{3} \\
\text{Riem. sum: } f(\frac{2}{3}) \cdot \frac{2}{3} + f(\frac{4}{3}) \cdot \frac{2}{3} + f(2) \cdot \frac{2}{3} \\
= (3(\frac{2}{3})^2+4) \cdot \frac{2}{3} + (3(\frac{4}{3})^2+4) \cdot \frac{2}{3} + (3(2)^2+4) \cdot \frac{2}{3}
\]

(b) [5 pts] Is your Riemann sum above an over- or under-estimate of the integral \( \int_0^2 f(x) \, dx \)?

It's an over-estimate because \( f \) is increasing on \([0, 2]\)

or: \ldots because the rectangles above contain more area than the region under the curve.

(c) [5 pts] Use summation (sigma) notation to write an expression for a Riemann sum for \( f \) on the interval \([0, 2]\) using \( n \) subintervals of equal width and using right endpoints on each subinterval. You do not have to work out the value of the sum, but your sum should involve only \( \sum_{k=1}^{n} \), the variables \( k \) and \( n \), and numbers.

\[
\Delta x = \frac{2}{n} \\
c_k = \frac{2k}{n} \\
f(c_k) = 3\left(\frac{2k}{n}\right)^2 + 4 = 3\left(\frac{2k^2}{n^2}\right) + 4
\]

\[
\text{Riem. sum: } \sum_{k=1}^{n} \left[ 3\left(\frac{2k^2}{n^2}\right) + 4 \right] \cdot \frac{2}{n} = \sum_{k=1}^{n} \left( \frac{12k^2}{n^2} + 4 \right) \cdot \frac{2}{n}
\]

= \sum_{k=1}^{n} \left( \frac{24k^2}{n^2} + \frac{8}{n} \right)
10. Use the values of the given definite integrals to determine the quantities below.

\[
\int_1^7 f(x) \, dx = -8, \quad \int_3^7 f(x) \, dx = 12, \quad \int_1^7 g(x) \, dx = 9
\]

(a) [5 pts] \[\int_1^7 (2f(x) - 5g(x)) \, dx\]

\[
= 2(-8) - 5(9) \\
= -16 - 45 \\
= -61
\]

(b) [5 pts] \[\int_1^3 f(x) \, dx = -20\]

(c) [5 pts] \[\int_1^7 (g(t) - t^2) \, dt\]

\[
= \int_1^7 g(t) \, dt - \int_1^7 t^2 \, dt \\
= 9 - \left[\frac{1}{3}t^3\right]_1^7 \\
= 9 - \left(\frac{1}{3} \cdot 7^3 - \frac{1}{3} \cdot 1^3\right) \quad \text{ok final answer} \\
= \frac{27}{3} - \frac{343}{3} + \frac{1}{3} = \frac{-315}{3} = 105
\]
11. The charts below contain information about a function \( f \) and its derivative. Assume that \( f \) is differentiable on \([-2,1]\). Use the charts to answer the questions that follow.

\[
\begin{array}{c|c|c|c|c|c|c}
 x & -2 & -1 & 0 & 1 \\
 f(x) & 3 & 2 & 0 & -1 \\
 f'(x) & -\frac{1}{8} & -\frac{1}{3} & -1 & 0 \\
\end{array}
\]

(a) [5 pts] Determine the linearization of \( f \) at \( x = -1 \).

\[
L(x) = f(-1) + f'(-1)(x + 1) \\
= \left(2 + \frac{1}{3}(-1 + 1)\right) \\
= 2 - \frac{1}{3} - \frac{1}{3}x \\
= \frac{5}{3} - \frac{1}{3}x
\]

(b) [5 pts] Use your linearization above to estimate the value of \( f(-1.5) \).

\[
f(-1.5) \approx L(-1.5) = \left(2 - \frac{1}{3}(-1.5 + 1)\right) \\
= 2 - \frac{1}{3}(\frac{1}{2}) \\
= 2 + \frac{1}{6} \\
= \frac{13}{6} \\
= 2.16
\]

(c) [5 pts] Suppose you also know that \( f' \) is continuous on \([-2,1]\). Explain why the graph of \( f \) must have an inflection point somewhere in the interval \([-2,1]\).

Based on the chart for \( f' \), there must be an inflection point in \([-2,1]\) since \( f' \) changes from decreasing to increasing (at least once).
12. [15 pts] A diesel truck develops an oil leak. The oil drips onto the dry ground in the shape of a circular puddle. Assuming that the leak begins at time \( t = 0 \) and that the radius of the oil slick increases at a constant rate of .05 meters per minute, determine the rate of change of the area of the puddle 4 minutes after the leak begins.

\[
\text{goal: } \frac{dA}{dt}
\]

\[
A = \pi r^2
\]

\[
\frac{dA}{dt} = 2\pi r \frac{dr}{dt}
\]

\[
\frac{dr}{dt} = 0.05 \text{ meters/minute}
\]

\[
r = \frac{0.05 \text{ meters}}{1 \text{ min}} \times 4 \text{ minutes} = 0.2 \text{ m}
\]

since \( \frac{dr}{dt} \) is constant

\[
\frac{dA}{dt} = 2\pi (0.2)(0.05) \text{ square meters/minute}
\]

\[
= 0.02\pi \text{ square meters/minute}
\]

\[
\approx 0.06283 \text{ square meters/minute}
\]
13. A landscape designer plans to construct a rectangular garden whose area is 2000 square meters. One side will consist of a wrought iron fence which costs $90 per meter. The remaining three sides will be constructed from chain link fence costing $25 per meter.

(a) [15 pts] Determine a function for the total cost $C(x)$ of the garden, where $x$ is the length of wrought iron fence used (in meters).

\[
C(x) = 90x + 25\left(\frac{2000}{x}\right) + 25x + 25\left(\frac{2000}{x}\right)
\]

(b) [15 pts] What dimensions of the garden will minimize the total cost? Use calculus techniques to show that the dimensions result in the minimum possible cost.

\[
\begin{align*}
\text{domain: } & (0, \infty) \\
C'(x) &= 115 - \frac{100,000}{x^2} \\
C''(x) &= \frac{200,000}{x^3}
\end{align*}
\]

\[
\begin{align*}
C'(x) &= 0: \\
115 &= \frac{100,000}{x^2} \\
x^2 &= \frac{100,000}{115} \\
x &= \sqrt{\frac{20,000}{23}} = 100 \cdot \sqrt{\frac{2}{23}}
\end{align*}
\]

Since there is only one critical number in $(0, \infty)$, the local min is an absolute min. The dimensions are $x = 100 \cdot \sqrt{\frac{2}{23}} \approx 29.4884$ meters and $y = \frac{2000}{100 \sqrt{\frac{2}{23}}} \approx 67.8233$ meters.

Note: The first derivative test is also fine to use; see the last page.
14. [10 pts] Let \( y = \ln(x) \). Show that \( \frac{dy}{dx} = \frac{1}{x} \) by solving the equation \( y = \ln(x) \) for \( x \) and then using implicit differentiation. Your final answer should be \( \frac{dy}{dx} \), given as a function of \( x \).

\[
\begin{align*}
y &= \ln(x) \\
e^y &= x & \text{(solve for } x) \\
e^y \frac{dy}{dx} &= 1 & \text{(differentiate both sides with respect to } x) \\
\frac{dy}{dx} &= \frac{1}{e^y} & \text{(solve for } \frac{dy}{dx}) \\
\frac{dy}{dx} &= \frac{1}{x} & \text{(sub in } x \text{ for } e^y \text{ since } e^y = x) 
\end{align*}
\]
15. Information about a function, $f$, and its derivative is given below. Use the information to answer the questions that follow.

Information about $f$:

$$\lim_{x \to 0^+} f(x) = 2,$$
$$\lim_{x \to 2^-} f(x) = 1,$$

This is the graph of the derivative of $f$:

(a) [5 pts] Make a rough sketch of the graph of $y = f(x)$. (Hint: Think about slopes.)

(b) [5 pts] Determine $\lim_{x \to 1^-} f(x)$.

(c) [5 pts] Determine $\lim_{x \to 1^+} f(x)$.

0
# 13 - 1st derivative test

\[ C'(x) = 115 - \frac{100,000}{x^2} = \frac{115x^2 - 100,000}{x^2} \]

Critical number: \( x = 100 \sqrt{\frac{2}{23}} \approx 29.49 \)

<table>
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<tr>
<th>( x )</th>
<th>( C'(x) )</th>
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<tbody>
<tr>
<td>( (0, 100 \sqrt{\frac{2}{23}}) )</td>
<td>( \Theta )</td>
</tr>
<tr>
<td>( (100 \sqrt{\frac{2}{23}}, \infty) )</td>
<td>( \Theta )</td>
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</table>

Therefore \( C(x) \) has a local min at \( x = 100 \sqrt{\frac{2}{23}} \)

Since there is only one critical number in \( (0, \infty) \), the local min is an absolute min. The dimensions are \( x = 100 \sqrt{\frac{2}{23}} \approx 29.4984 \) meters and \( y = \frac{200}{100 \sqrt{\frac{2}{23}}} \approx 67.8233 \) meters.