1) Let $A$ and $B$ be disjoint small round open disks in $S^2$ and let $h$ be a homeomorphism from $A$ to $B$. Let $X = S^2 / \sim$ have the quotient topology where $a \sim h(a)$ for all $a \in A$.

   a) Is $X$ Hausdorff?

   b) Is $X$ connected?

2) Let $X$ be the union of a torus and a disk as shown below. Compute the homology of $X$.

   \[
   \begin{array}{c}
   \includegraphics[width=2cm]{torus}\n   \end{array}
   \]

3) Let $T$ be a torus minus a small round open disk. Let $\alpha = \partial T$ and use covering spaces to show that $[\alpha] \neq 1 \in \pi_1(T, \ast)$.

4) State the homotopy lifting theorem for covering spaces and then use it to compute $\pi_1(S^1, \ast)$.

5) Let $X$ be an arbitrary subset of $\mathbb{R}$. Let $f : X \to \mathbb{R}$ be proper (this means $f^{-1}(\text{compact subset of } \mathbb{R}) = \text{compact subset of } X$). Show that the graph of $f$ is a closed subset of $\mathbb{R}^2$.

6) Let $f : X \to X$ be continuous.
   a) If $X = S^1 \vee S^1$ must $f$ have a fixed point?
   b) If $X = \bigvee_{\text{interior point of each}}$ (the wedge product of two closed intervals along an interior point of each) must $f$ have a fixed point?

7) Let $S^1 = \{ \bar{u} \in \mathbb{R}^2 \mid |\bar{u}| = 1 \}$. Suppose $f : S^1 \to \mathbb{R}^2 - 0$ is continuous and $\bar{u} \cdot f(\bar{u}) > 0$ for all $\bar{u} \in S^1$. Prove that $f$ does not extend to a function from $D^2$ into $\mathbb{R}^2 - 0$.

8) Let $A$ be the union of two disjoint circles contained in $\mathbb{R}^3 \subset \mathbb{R}^3 \cup \{ \infty \} = S^3$ as indicated. Compute the homology of $\mathbb{R}^3 - A$. 

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