1. Prove that a continuous bijection from a compact space to a Hausdorff space is a homeomorphism.

2. Let $X$ and $Y$ be topological spaces. Do one of the following.
   (a) Prove if $X$ and $Y$ are path connected then $X \times Y$ is path connected.
   (b) Prove if $X$ and $Y$ are compact then $X \times Y$ is compact.

3. Let $A$ be a closed subspace of the regular Hausdorff space $X$ and $X \overset{p}{\rightarrow} X/\sim$ be the natural projection where $\sim$ is the equivalence relation defined by $a \sim b$ if $a$ and $b$ are elements of $A$. Prove $X/\sim$ is Hausdorff if the topology on $X/\sim$ is the quotient topology induced from $p$.

4. Classify all covering spaces of $P \times P$ where $P$ is a 2-dimensional real projective space.

5. Let $X$ be homeomorphic to a 2-dimensional sphere, $Y$ be homeomorphic to a 2-dimensional torus, and $Z$ be the one point union of $X$ and $Y$.
   (a) Compute the fundamental group of $Z$.
   (b) Compute $H_*(Z,\mathbb{Z})$.

6. Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $f$ the induced map on the 2-dimensional torus $T$ making the diagram $\begin{array}{ccc} \mathbb{R}^2 & \overset{A}{\rightarrow} & \mathbb{R}^2 \\ \downarrow{p} & & \downarrow{p} \\ T & \overset{f}{\rightarrow} & T \end{array}$ commute, where $p(x, y) = (e^{2\pi ix}, e^{2\pi iy})$ is the natural universal covering map of $T = S^1 \times S^1$.
   (a) Prove $f$ is a homeomorphism.
   (b) Prove or disprove: $f$ has a fixed point.

7. Prove there does not exist a retraction of the 3-dimensional sphere $S^3$ onto a subspace that is homeomorphic to a closed connected 2-manifold.

8. Prove there does not exist a continuous map $S^n \overset{f}{\rightarrow} S^{n-1}$ such that $f(-x) = -f(x)$ for all $x \in S^n$ where $S^n \subseteq \mathbb{R}^{n+1}$ is the unit sphere and $n \geq 1$. 