Study Guide for Algebra Qualifying Exam

Group Theory

subgroups and quotient groups
Lagrange's Theorem
fundamental homomorphism theorems
group actions with applications to the structure of groups such as
    the Sylow Theorems
group constructions such as:
    direct and semi-direct products
structures of special types of groups such as:
    p-groups
    dihedral, symmetric and alternating groups, cycle decompositions
    the simplicity of A_n, for $n \geq 5$
free groups, generators and relations
solvable groups

References: [1,3,4]

Linear Algebra

determinants
eigenvalues and eigenvectors
Cayley-Hamilton Theorem
canonical forms for matrices
linear groups (GL_n , SL_n, O_n, U_n)
dual spaces, dual bases, induced dual map, double duals
finite-dimensional spectral theorem

References: [1,2,4]

Foundations

Zorn's Lemma and its uses in various existence theorems such as that of a basis for a vector space or existence of maximal ideals.

References: [1,3,4]

Theory of Rings and Modules

basic properties of ideals and quotient rings
fundamental homomorphism theorems for rings and modules
characterizations and properties of special domains such as:
    Euclidean implies PID implies UFD
classification of finitely generated modules over PIDs with emphasis on Euclidean domains
applications to the structure of:
    finitely generated abelian groups
    canonical forms of matrices

References: [1,3,4]

Field Theory

algebraic extensions of fields
fundamental theorem of Galois theory
properties of finite fields
separable extensions
computations of Galois groups of polynomials of small degree and cyclotomic polynomials
solvability of polynomials by radicals

References: [1,3,4]

As a general rule, students are responsible for knowing both the theory (proofs) and practical applications (e.g. how to find the Jordan or rational canonical form of a given matrix, or the Galois group of a given polynomial) of the topics mentioned. A supplement to this study guide is available at:
http://www.math.uga.edu/sites/default/files/PDFs/Graduate/QualsStudyGuides/AlgebraPhDqualremarks.pdf

References

[1] David Dummit and Richard Foote, *Abstract Algebra*, Wiley, 2003.
[2] Kenneth Hoffman and Ray Kunze, *Linear Algebra*, Prentice-Hall, 1971.
[3] Thomas W. Hungerford, *Algebra*, Springer, 1974.
[4] Roy Smith, *Algebra Course Notes* (843-1 through 845-3), http://www.math.uga.edu/~roy/, 1996.

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