GRADUATE COURSE IN ANALYSIS

(1) Banach spaces, Hilbert spaces and $L^p$ spaces
   • Banach spaces: definitions, linear functionals and dual of a Banach space.
   • Hahn-Banach Theorem: extension of linear functionals and separation of convex sets.
   • Weak* topology on dual Banach spaces: strong, weak and weak* topologies, the Banach-Alaoglu Theorem.
   • $L^p$ spaces: Hölder and Minkowski inequalities, completeness and dual of $L^p$, the case of $L^\infty$.

(2) The theory of Distributions.
   • Distributions: definitions, operations, localization and convergence.
   • Tempered distributions: definitions, properties and the Fourier transform.
   • Fundamental solutions to partial differential equations with constant coefficients.
   • Regularity: weak derivatives, Sobolev spaces, approximates to the identity, Sobolev inequalities and Embedding theorems

(3) The Fourier transform
   • The Schwartz space. The Fourier transform on $\mathcal{S}(\mathbb{R})$.
   • The Fourier inversion formula and the Plancherel Theorem.
   • Applications to partial differential equations.
   • The Poisson summation formula and the Heisenberg uncertainty principle.