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TEAM ROUND / 45 MIN / 150 POINTS

No calculators are allowed on this test. You do not have to provide proofs; only the answers matter. Each problem is worth 50 points, for a total of 150 points.

For problem 3, the answer should be an exact expression, such as $\pi/2$, $\sqrt{3} + 1$, $8/3$, etc. No approximate answers will be accepted.

Problem 1. (Five secret numbers) Suppose there are 5 numbers whose pairwise sums are

5, 9, 20, 24, 31, 35, 39, 42, 46, 61

What are the original 5 numbers? Write them in increasing order.

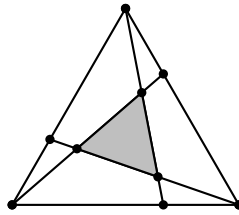
Problem 2. (The last man standing) n people stand in a circle. Then, every second person is excluded until only one is left. For example, with 10 people, the order of exclusion is as follows:

2, 4, 6, 8, 10, 3, 7, 1, 9

so the last remaining person is number 5.

Now start with 2005 people. Who will be the last person standing?

Problem 3. (Two triangles) In a triangle ABC , vertices are connected to the points A' , B' , C' which divide the corresponding sides with the ratio 2 to 1, as in the picture, to form a small triangle KLM in the center. What is the ratio of area of ABC to the area of KLM ? (The answer must be greater than 1.)



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