No calculators are allowed on this test. You do not have to provide proofs; only the answers matter. Each problem is worth 50 points, for a total of 150 points.

Problem 1. (Color the cube) In how many ways can one color the six faces of the cube using two red squares, two blue squares and two white squares? (Two colorings are considered to be the same if one can be rotated into the other.)

Problem 2. (Kissing spheres) 4 spheres of radius 1 are placed so that each touches the other three. What is the radius of the smallest sphere that contains all 4 spheres?

Problem 3. (Happy squares) In how many ways can one put numbers 1 through 9 in a 3-by-3 square in such a way that numbers in every row are increasing (from left to right) and numbers in every column are increasing (from top to bottom).

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