



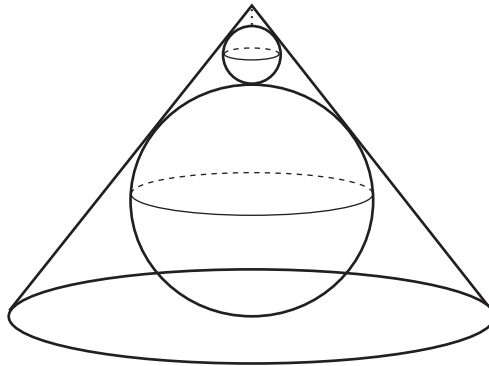
Sponsored by: UGA Math Department and UGA Math Club

TEAM ROUND / 45 MIN / 210 POINTS  
October 13, 2007

**No calculators are allowed on this test.** You do not have to provide proofs; only the answers matter. Each problem is worth 70 points, for a total of 210 points.

**Problem 1. (Spheres)** An infinite sequence of spheres is inscribed in a right circular cone with base 2, with the largest having radius 1, as shown. Find the volume of that portion of the cone lying outside all the spheres.

Give the answer as a fraction, possibly involving  $\pi$ ,  $\sqrt{2}$ , etc. In other words, give the exact answer, not a decimal approximation.



**Problem 2. (Sudoku)** A  $4 \times 4$  Sudoku puzzle is a  $4 \times 4$ -square filled with numbers 1,2,3,4 so that the numbers in each row, column, and each of the four  $2 \times 2$ -square do not repeat. (You may already be familiar with a more complicated but similar  $9 \times 9$  Sudoku puzzle.)

1	2	3	4
3	4	1	2
2	1	4	3
4	3	2	1

How many  $4 \times 4$  puzzles are there? (Puzzles that are obtained from each other by rotation, reflection, etc. count as *different* puzzles.)

**Problem 3. (Coin flipping)** A person is flipping a coin many times. With probability  $1/3$  each of the following can happen on each flip: heads comes up, tails comes up, or she gets bored and decides to stop. What is the probability that, after the game has stopped, there were never two heads in a row?

**Authors.** (1) was written by Ted Shifrin, (2) and (3) by Valery Alexeev and Boris Alexeev.

RETURN THIS SHEET

**Team ID:**

Team name:

Answer 1:

Answer 2:

Answer 3: