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TEAM ROUND / 45 MIN / 210 POINTS
October 13, 2007

No calculators are allowed on this test. You do not have to provide proofs; only the answers matter. Each problem is worth 70 points, for a total of 210 points.

Problem 1. (Platonic solids and plane tilings) How many triples of positive integers (p, q, r) are there such that

- (a) Each of p , q , and r is at least 2,
- (b) At most one of p , q , and r equals 2, and
- (c)

$$\frac{1}{p} + \frac{1}{q} + \frac{1}{r} \geq 1?$$

Note that in this problem, triples are *unordered*; for example, (p, q, r) and (q, p, r) count as different triples if $p \neq q$.

Problem 2. (M-triples) How many triples of *positive integers* (a, b, c) are there with $a \leq b \leq c \leq 100$ that satisfy the equation $a^2 + b^2 + c^2 = 3abc$?

Note that in this problem, triples are *ordered*; we insist that $a \leq b \leq c$.

Problem 3. (Holy polyhedron) There exists a (non-convex) polyhedron with exactly one hole, such that every pair of faces shares exactly one edge. How many vertices does this polyhedron have?

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Team ID:

Team name:

Answer 1:

Answer 2:

Answer 3: